

1. For the graph of $f(x) = x^4 - 12x^3 + 36x^2 + 4x - 12$, find the equation of the line that is tangent to the graph of $f(x)$ at two points. **Use calculus and algebra to show that this tangent line is the only one that is tangent twice to the graph of $f(x)$.** This means solving the equation

$$\frac{f(b) - f(a)}{b - a} = f'(b) = f'(a)$$

where a and b are the two distinct points of tangency. (Note that this can be written as three separate equations, namely

$$\frac{f(b) - f(a)}{b - a} = f'(b)$$

$$\frac{f(b) - f(a)}{b - a} = f'(a)$$

$$f'(b) = f'(a)$$

Noting you may assume that $b - a \neq 0$, add the first two equations together, simplify and factor everything to get an expression for a in terms of b . Then substitute this expression for a into the right-hand side of the last equation and solve for b . Lastly, use your expression for a in terms of b to find a , and check that the tangent lines to $f(x)$ at $x = a$ and $x = b$ are the same.)