Final Exam

Name: ______________________

Except for the last problem, each problem is worth 20 points. Numerical estimates are unacceptable unless specifically requested; for full credit you must show all your work and use the indicated methods. Work at least ten problems; you may work one additional problem for extra credit. If you work more than eleven problems, I will only grade the first eleven.

1. (20 pts) Use the definition of the limit to show that

$$\lim_{x \to -5} 2x + 17 = 7$$

2. (20 pts) Find

$$\lim_{x\to\infty} \frac{5x^3 - 3}{4x^3 + 2x^2 - 3x + 7}$$

(a) using l'hôpital's rule

(b) without using l'hôpital's rule

3. (20 pts) Draw a graph of a function $f(x)$ that satisfies the following conditions:

(i) $\lim_{x \to -1} f(x) = 3$

(ii) $\lim_{x \to 0^-} f(x) = 0$

(iii) $\lim_{x \to 0^+} f(x) = \infty$

(iv) $\lim_{x \to 3} f(x)$ DNE

(v) $\lim_{x \to 3^+} f(x) = -2$

(vi) $\lim_{x \to \infty} f(x) = -4$

(vii) $\lim_{x \to -\infty} f(x) = -2$

(viii) $f(x)$ is continuous everywhere except $x = -1, 0, 3$

4. (20 pts) Use the definition of the derivative to find

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(x + \sqrt{x}\right)$$

5. (20 pts) Use the definition of the derivative to prove the constant multiple rule for derivatives:

$$\frac{\mathrm{d}}{\mathrm{d}x} c f(x) = c \frac{\mathrm{d}}{\mathrm{d}x} f(x)$$

(where $f(x)$ is assumed to be differentiable).

6. (20 pts) Find

$$\lim_{x \to 1} (\ln x)^{(\ln x)}$$

7. (20 pts) Find

$$\frac{\mathrm{d}}{\mathrm{d}x}\,(\ln x)^{(\ln x)}$$

8. (20 pts) Suppose the position of a particle on the x-axis after t seconds is given by $f(t) = 3t^5 - 320t^3$. Determine when the particle is speeding up and when it is slowing down.

9. (20 pts) Prove that

$$\frac{\mathrm{d}}{\mathrm{d}x} \sin x = \cos x$$

You may assume the following:

- $$\sin(x+h) = \sin x \cos h + \cos x \sin h$$

- $$\lim_{h \to 0} \frac{\sin h}{h} = 1$$

- $$\lim_{h \to 0} \frac{\cos h - 1}{h} = 0$$

10. (20 pts) Find the equation of the tangent line to the graph of $f(x) = \cos x$ when $x = \frac{3\pi}{4}$.

11. (20 pts) Show that

$$\lim_{x\to 0^+} \sqrt{x}\cos\frac{\pi}{x} = 0$$

=0

12. (20 pts) Find the following derivatives:

(i) $\frac{\mathrm{d}}{\mathrm{d}x} 2^x$

(ii) $\frac{\mathrm{d}}{\mathrm{d}x} \sin \sqrt{x}$

(iii) $\frac{\mathrm{d}}{\mathrm{d}x} \frac{\ln x}{\tan x + \pi x}$

(iv) $\frac{\mathrm{d}}{\mathrm{d}x} \sqrt{x} \sin x$

13. (20 pts) Find the equation of the tangent line to the graph of $r = e^{\theta}$ when $\theta = \pi$.

14. (20 pts) Use local linear approximation to estimate $\sqrt[3]{100}$ Explain why your approximation is an overestimate or an underestimate by referring to a graph or a second derivative.

15. (20 pts) Determine how quickly the dimensions of a 4in by 8in rectangle are increasing if the area is increasing by $40\text{in}^2/\text{s}$ and the perimeter is increasing by 12in/s.

16. (20 pts) Suppose a rectangular box with an open top is to be constructed with a base twice as long as it is wide, and must have a capacity of 5ft^3. Find the dimensions of the box that minimize the amount of material used in its construction.

17. (40 pts) Suppose a time capsule is to be constructed in the shape of a cylinder with two identical conical caps on the ends. If the radius of the capsule is 1in and it must be designed to hold 20in^3, determine the dimensions which minimize its surface area (recall that the surface area of the side of a cone is given by $V = \frac{1}{3}\pi r^2 h$).