Practice Exam 3

1. $\frac{d}{dx} \sin e^{\sin x}$

$f(x) = \sin x \qquad f'(x) = \cos x$

$g(x) = e^{\sin x} \qquad g'(x) = \cos x e^{\sin x}$

$h(x) = e^x \qquad h'(x) = e^x$

$k(x) = \sin x \qquad k'(x) = \cos x$

$g'(x) = h'(k(x))\, k'(x)$

$= e^{\sin x} \cos x$

$f'(g(x))g'(x) = \cos(e^{\sin x}) \cos x e^{\sin x}$

2. $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{2t \sec^2 t^2}{(\ln 2) 2^t}$

$= \frac{4 \sec^2 4}{(\ln 2) 4} = \frac{\sec^2 4}{\ln 2}$

$y - \tan 4 = \frac{4 \sec^2 4}{\ln 2}(x-4)$

$y = \frac{\sec^2 4}{\ln 2} x + \frac{\ln 2 \tan 4 - 4\sec^2 4}{\ln 2}$

3. $\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{\frac{d}{d\theta}(\sin\theta \cos\theta)}{\frac{d}{d\theta}(\cos^2\theta)}$

$= \frac{\cos^2\theta - \sin^2\theta}{-2\cos\theta \sin\theta}$

$dy/d\theta = 1$

$dx/d\theta = 0$

vertical tangent $\quad x = -1$

4 $e^y + xe^y y' = y'\sin x + y\cos x$

$$xe^y y' - y'\sin x = y\cos x - e^y$$
$$y'(xe^y - \sin x) = y\cos x - e^y$$
$$y' = \frac{y\cos x - e^y}{xe^y - \sin x}$$

5. $y = \sec^{-1} x$

$$f^{-1}(x) = \sec^{-1} x$$
$$f(x) = \sec x = 1/\cos x$$
$$f'(x) = \sec x \tan x$$
$$= \frac{+\sin x}{\cos^2 x} = \sec x \tan x$$

$$(f^{-1}(x))' = \frac{1}{f'(f^{-1}(x))}$$

$\tan^2\alpha + 1 = \sec^2\alpha$
$\tan\alpha = \sqrt{\sec^2\alpha - 1}$

$$= \frac{1}{\sec\sec^{-1}x \tan\sec^{-1}x}$$
$$= \frac{1}{x\sqrt{\sec^2\sec^{-1}x - 1}}$$

$$\frac{d}{dx}\sec^{-1}x = \frac{1}{x\sqrt{x^2-1}}$$

6. $= \frac{d}{dx}\left(2\ln x - \ln(x+1)\right)$

$$= \frac{2}{x} - \frac{1}{x+1}$$

7 $y = \sqrt{x}^{\sqrt[3]{x}}$

$\ln y = \sqrt[3]{x} \ln\sqrt{x}$

$= \frac{\sqrt[3]{x}}{2} \ln x$

$\frac{y'}{y} = \frac{1}{6\sqrt[3]{x^2}} \ln x + \frac{\sqrt[3]{x}}{2x}$

$= \frac{\ln x + 3}{6\sqrt[3]{x^2}}$

$y' = \sqrt{x}^{\sqrt[3]{x}} \left(\frac{\ln x + 3}{6\sqrt[3]{x^2}}\right)$

8 $f(x) = \sin x$ $\quad f(a) = 0$

$a = \pi$ $\quad f'(a) = -1$

$f'(x) = \cos x$

$f(x) \approx f'(a)(x-a) + f(a)$

$= -x + \pi$

$f(3) \approx \pi - 3$

$= 0.14159$

9. $r = 1/\sqrt{\theta} = \theta^{-1/2}$

$\frac{dr}{dt} = \frac{-1}{2\sqrt{\theta^3}} \frac{d\theta}{dt}$ $\quad \frac{d\theta}{dt} = 12\pi$ rad/min

$\theta = 1/r^2$

$= 1/(1/2)^2$

$= 1/(1/4)$

$= 4$

$= \frac{-1}{2\sqrt{4^3}} 12\pi$

$= \frac{-12\pi}{16} \approx -2.356$

Moving towards the center at -2.356 furlongs/min

10 $H(t) = 4-4e^{-t}\cos t$

$H'(t) = 4e^{-t}\cos t + 4e^{-t}\sin t$

$0 = 4e^{-t}(\sin t + \cos t)$

$\sin t = -\cos t$

$\tan t = -1$

$t = \frac{3\pi}{4} + k\pi, k \in \mathbb{Z}$

$\text{dom } H = [0, \infty)$

g min $H(0) = 0$

g max $H(3\pi/4) = 4.268$

$H(7\pi/4) = 3.988$

$H(11\pi/4) = 4.0005$

$H(15\pi/4) = 3.99998$

$H(19\pi/4) = 4.0000009$

$H(23\pi/4) = 3.99999996$

getting closer to 4

$\lim_{t\to\infty} H(t) = 4$