Name: _____

Each problem is worth the indicated number of points. Work five of the first six problems and one of the last two; you may solve one additional problem for extra credit (if you work all eight, I will only grade the first seven). Show all your work for full credit (excluding arithmetic); numerical or graphical estimates are unacceptable unless specifically requested.

1. (15 pts) Find the following derivatives:

   (a) $\frac{\mathrm{d}}{\mathrm{d}x} e^{\tan x}$

   (b) $\frac{\mathrm{d}}{\mathrm{d}x} \log_5 x$

   (c) $\frac{\mathrm{d}}{\mathrm{d}x} (\cos x)^{(\sin x)}$

   (d) $\frac{\mathrm{d}}{\mathrm{d}x} \sqrt[4]{\tan x} \csc^{-1} 3^x$

2. (15 pts) Find the slope of the tangent line to the graph of the parametric equation
equation
$$\begin{cases} x &=& t^2 - 3 \\ y &=& 2t + 2 \end{cases}$$
when $t = 4$

3. (15 pts) State the differentiation rule for $\cot^{-1} x$, and prove this rule using implicit differentiation, trigonometric identities, and the fact that $\frac{d}{dx}\cot x = -\csc^2 x$.

4. (15 pts) Find the y' if

$$\tan(x+y) = \cot(x+y)$$

5. (15 pts) Use local linear approximation to estimate $\tan 0.75$. Tell whether this is an underestimate or an overestimate, and explain why (hint: determine the concavity of the graph using a second derivative). For this problem only, <u>you must show all arithmetic performed by hand, and you will receive no credit for any work done using a calculator</u>. You may leave your final answer in the form of a fraction. (Hint: to find a in the linear approximation formula consider fractional multiples of π close to 0.75 that have easy to calculate values for $\tan a$. You may approximate π by 3.14.)

6. (15 pts) Find all global maxima and minima of

$$f(x) = x^5 - 405x$$

on $[0, 5]$.

7. (25 pts) Suppose a paper lantern hanging on a hook six feet above a porch is knocked loose by the wind. The lantern takes four seconds to fall to the porch at constant velocity, in which time it floats horizontally one foot closer to a two and a half foot tall chair placed four feet away from a point directly beneath the hook. Determine the length of the chair's shadow and how it is changing when the lantern is half-way down.

8. (25 pts) Determine the growth rate of the radius of a sphere when its surface area is increasing by $25\text{in}^2/\text{min}$ and its radius is 4in.