Exam 2

Name: _____

Each problem is worth the indicated number of points. Work five of the first six problems and one of the last two; you may solve one additional problem for extra credit (if you work all eight, I will only grade the first seven). Show all your work for full credit (excluding arithmetic); numerical or graphical estimates are unacceptable unless specifically requested.

1. (15 pts) Find the following derivatives:

   (a) $\frac{\mathrm{d}}{\mathrm{d}x} e^{\sqrt{x}}$

   (b) $\frac{\mathrm{d}}{\mathrm{d}x} \ln x$

   (c) $\frac{\mathrm{d}}{\mathrm{d}x} x^x$

   (d) $\frac{\mathrm{d}}{\mathrm{d}x} \frac{\sqrt{\cos x}}{\log_2 \cot x}$

2. (15 pts) Find where the slope of the tangent line to the graph of the polar equation

$$r = \sin\theta, 0 \leq \theta \leq 2\pi$$

is horizontal and where it is vertical.

3. (15 pts) State the differentiation rule for $\csc^{-1} x$, and prove this rule using implicit differentiation, trigonometric identities, and the fact that $\frac{\mathrm{d}}{\mathrm{d}x} \csc x = -\csc x \cot x$. (Note: when you need to take a square root, you may assume that the positive branch is correct without providing any additional justification.)

4. (15 pts) Find the y' if

$$(x+y)^3 = (x-y)^3$$

5. (15 pts) Use local linear approximation to estimate $\sqrt[4]{250}$. Tell whether this is an underestimate or an overestimate, and explain why (hint: determine the concavity of the graph using a second derivative). For this problem only, you must show all arithmetic performed by hand, and you will receive no credit for any work done using a calculator. You may leave your final answer in the form of a fraction. (Hint: to find a in the linear approximation formula, calculate 4^{th} powers of integers until you find one close to 250.)

6. (15 pts) Find all local and global maxima and minima of

$$f(x) = 3x^4 - 4x^3 - 36x^2$$

on $[-2, 2]$.

7. (25 pts) Suppose a stock broker convicted of insider trading violates his house arrest and travels north on foot at a rate of one block/minute. Meanwhile, his parole officer tracks the stock broker via the GPS tracking unit in his ankle bracelet. The parole officer starts out 30 blocks south and 3 blocks west of the stock broker and travels east in the squad car at a rate of 5 blocks/minute. Determine whether they are getting closer together or further apart, and how quickly.

8. (25 pts) Determine the growth rate of the radius of a cylinder when its volume is growing by $50\text{in}^3/\text{min}$ and its surface area (including the end caps) is growing by $20\text{in}^2/\text{min}$, assuming its height and diameter are always equal. (Hint: first find the radius.)