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Name: KGY

Each problem is worth 15 points. Show all your work for full credit; numerical or graphical estimates are unacceptable unless specifically requested.

1. Find the following:

$\frac{d}{dx}\left[e^x + x^e + 3e^2 + 3x^2 + \frac{x}{e} + \frac{e}{x}\right]$

$= e^x + ex^{e-1} + 6x + \frac{1}{e} - \frac{e}{x^2}$

$\frac{d}{dx}[\sin x \cos x]$

$= \cos^2 x - \sin^2 x$

$\frac{d}{dx}\left[\sqrt{x}\,(x - \sqrt[3]{x})\right]$

$= \frac{d}{dx}\left(x^{3/2} - x^{5/6}\right)$

$= \frac{3}{2}x^{1/2} - \frac{5}{6}x^{-1/6}$

$= \frac{3}{2}\sqrt{x} - \frac{5}{6\sqrt[6]{x}}$

$\frac{d}{dx}\left[\frac{\sqrt{x}}{e^x+1}\right]$

$= \frac{\frac{1}{2\sqrt{x}}(e^x+1) - \sqrt{x}\,e^x}{(e^x+1)^2}$

2. Suppose the air pressure in a tire (measured in PSI) after t minutes of pumping is given by

$$P(t) = \begin{cases} t^3 - 27t \overset{+54}{} & \text{if } 0 \le t \le 10 \\ 0 & \text{if } t > 10 \end{cases}$$

Find when the rate of pressure increase is speeding up and when it is slowing down.

$P'(t) = 3t^2 - 27$

$P''(t) = 6t$

$0 = 3t^2 - 27$

$= 3(t^2 - 9)$

$t = \pm 3$

$0 = 6t$

$t = 0$

	$P'(t)$	$P''(t)$	
$(0, 3)$	$-$	$+$	Slowing
$(3, 10)$	$+$	$+$	Speeding
$(10, \infty)$	0	0	Not changing

Pressure is speeding up from

3, 10 sec

Slowing from 0, 3 sec

3. Use the definition of the derivative to find

$$\frac{\mathrm{d}}{\mathrm{d}x}\frac{1}{\sqrt{x}}$$

$$= \lim_{h\to 0} \frac{\frac{1}{\sqrt{x+h}} - \frac{1}{\sqrt{x}}}{h} = \lim_{h\to 0} \frac{\sqrt{x} - \sqrt{x+h}}{h\sqrt{x}\sqrt{x+h}}$$

$$= \lim_{h\to 0} \frac{\sqrt{x} - \sqrt{x+h}}{h\sqrt{x}\sqrt{x+h}} \frac{\sqrt{x} + \sqrt{x+h}}{\sqrt{x} + \sqrt{x+h}}$$

$$= \lim_{h\to 0} \frac{\cancel{x} - (\cancel{x} + \cancel{h})}{\cancel{h}\sqrt{x}\sqrt{x+h}(\sqrt{x} + \sqrt{x+h})}$$

$$= \lim_{h\to 0} \frac{-1}{\sqrt{x}\sqrt{x+h}(\sqrt{x} + \sqrt{x+h})}$$

$$= \frac{-1}{2(\sqrt{x})^3} = \frac{-1}{2x^{3/2}} = \frac{-1}{2}x^{-3/2}$$

4. State the differentiation rule for $\csc x$, and prove this rule using the differentiation rules for $\sin x$ and $\cos x$.

$$\frac{d}{dx} \csc x = -\csc x \cot x$$

$$\frac{d}{dx} \csc x = \frac{d}{dx} \frac{1}{\sin x} = \frac{0(\sin x) - 1(\cos x)}{\sin^2 x}$$

$$= \frac{-\cos x}{\sin x} \frac{1}{\sin x} = -\cot x \csc x$$

$$= -\csc x \cot x$$

5. Find the tangent to the graph of $f(x) = e^x\sqrt{x}$ at $x = 3$.

$$f'(x) = e^x\sqrt{x} + \frac{e^x}{2\sqrt{x}}$$

$$f'(3) = e^3\sqrt{3} + \frac{e^3}{2\sqrt{3}}$$

$$f(3) = e^3(\sqrt{3})$$

$$\cancel{y - e^3\sqrt{3}}$$

$$y - e^3\sqrt{3} = \left(e^3\sqrt{3} + \frac{e^3}{2\sqrt{3}}\right)(x-3)$$

$$y - 34.789 \cong 40.587(x-3)$$

$$y \cong 40.587x - 86.973$$

6. Suppose Fred determines that his average cycling speed at a body weight of w pounds is given (in mph) by $T(w) = \frac{-1}{75}w^2 + \frac{68}{15}w - \frac{1108}{3}$. Find $T(165), T'(165)$, and $T''(165)$. Give units and interpret your answer.

$$T'(w) = \frac{-2}{75}w + \frac{68}{15}$$

$$T''(w) = \frac{-2}{75}$$

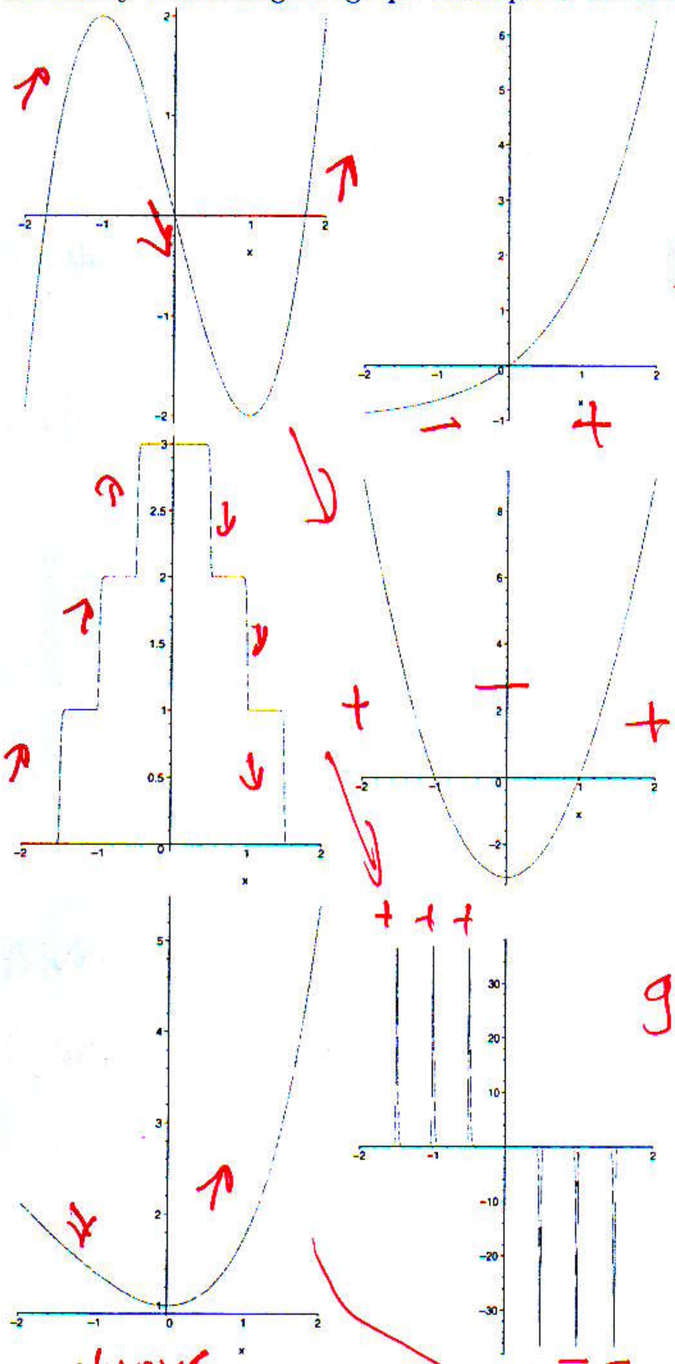
$T(165) = 15.67$ mph

$T'(165) = 0.133$ mph/lb

$T''(165) = -2/75 \approx -0.0267$ mph/lb^2

When Fred weighs 165 lbs, his avg. speed is ~~increa~~ 15.67 mph and increasing (improving) by 0.133 mph/lb, but the rate of ~~increase~~ improvement is slowing down by 0.0267 mph/lb^2

7. Match the graph of each function in the left column with the graph of its derivative in the right column. Give reasons for your choices that explain how the slope and concavity of the original graph correspond to features in the graph of the derivative.



f

g

no concavity

h

always cup

h'

always inc

f'

g'

f inc on $(-\infty,-1)\cup(1,\infty)$

f' + there

f dec on $(-1,1)$

f' neg there

f con on $(-\infty,0)$

f' dec there

f cup on $(0,\infty)$

f' inc there

1:34:35