Exam 1

Name: ______________________

Each problem is worth 15 points. Show all your work for full credit; numerical or graphical estimates are unacceptable unless specifically requested.

1. Graph each of the following sets of parametric equations:

   (a) $\begin{cases} x = \sec t \\ y = \tan^2 t \end{cases} \quad \left(-\frac{\pi}{2} \le t \le \frac{\pi}{2}\right)$

   (b) $\begin{cases} x = e^{-t} \\ y = e^{-2t} - 1 \end{cases}$

2. For the function $f(x) = \frac{x^4-16}{x^2+3x-10}$,

   (a) Find $\lim_{x\to 2} f(x)$ using the limit laws.

   (b) Check your answer from part 2a by approximating the limit with a table of values.

3. For the functions $f(x) = \frac{2|x|}{x} + 2$ and $g(x) = 3 - \frac{3|x|}{x}$, find the value of the limit or explain why it does not exist. For parts 3a, 3b, 3d, and 3e, you may use a graph to find the value of the limit.

   (a) $\lim_{x \to 0^+} f(x)$

   (b) $\lim_{x \to 0^-} f(x)$

   (c) $\lim_{x \to 0} f(x)$

   (d) $\lim_{x \to 0^+} g(x)$

   (e) $\lim_{x \to 0^-} g(x)$

   (f) $\lim_{x \to 0} g(x)$

   (g) $\lim_{x \to 0^+} (f + g)(x)$

   (h) $\lim_{x \to 0^-} (f + g)(x)$

   (i) $\lim_{x \to 0} (f + g)(x)$

   (j) $\lim_{x \to 0^+} (fg)(x)$

   (k) $\lim_{x \to 0^-} (fg)(x)$

   (l) $\lim_{x \to 0} (fg)(x)$

4. Show that

$$\lim_{x \to 0} \sin\left(\frac{\pi}{x}\right) \sin\left(\frac{x}{\pi}\right) = 0$$

5. For the function $f(x)$ and each of the points a listed below, determine whether $f(x)$ is continuous at $x = a$ and explain why:

$$f(x) = \begin{cases} \cos\left(\frac{\pi}{x}\right) & \text{if} \quad x < -1 \\ \frac{1}{x} & \text{if} \quad -1 \le x \le 1 \\ \sin\left(\frac{\pi}{x}\right) & \text{if} \quad x > 1 \end{cases}$$

(a) $a = -2$

(b) $a = -1$

(c) $a = 0$

(d) $a = 1$

(e) $a = 2$

6. Find

$$\lim_{x\to\infty} \frac{4x^3 - 7x}{\sqrt{16x^6 + 14}}$$

7. State the formal definition of the limit, and use the definition to prove that

$$\lim_{x\to -3} (-3x - 3) = 6$$

Illustrate your proof with a diagram showing $f(x)$, L, a, $L + \epsilon$, $L - \epsilon$, $a + \delta$, and $a - \delta$.

8. (Bonus) Find

$$\lim_{h\to 0} \frac{\sqrt[3]{27+h}-3}{h}$$