Name: KEY

Each problem is worth 25 points. Show all your work.

1. Find the exact value of $\csc\left(\cos^{-1}\left(\frac{60}{229}\right)\right)$

i.e. $\cos\theta = \frac{60}{229}$. Find $\csc\theta$

$\sin^2\theta + \cos^2\theta = 1$

$\sin\theta = \pm\sqrt{1-\cos^2\theta}$

$= \sqrt{1-\cos^2\theta}$

Since θ is in QI

$\sin\theta = \sqrt{1-\left(\frac{60}{229}\right)^2}$

$= \sqrt{1-\frac{3600}{52441}}$

$= \sqrt{\frac{52441-3600}{52441}}$

$= \sqrt{\frac{48841}{52441}} = \frac{221}{229}$

So $\csc\theta = \frac{1}{\sin\theta} = \frac{229}{221}$

2. Find the exact value of

Use $1/2$-angle id.

(a) $\cos\left(\frac{\pi}{24}\right) =$

$\cos\left(\frac{\pi/12}{2}\right) =$

$\cos\left(\frac{\pi/6}{2}/2\right) = \pm\sqrt{\frac{1+\cos\left(\frac{\pi/6}{2}\right)}{2}} = \pm\sqrt{\frac{1\pm\sqrt{\frac{\cos(\pi/6)+1}{2}}}{2}}$

$= \sqrt{\frac{1+\sqrt{\frac{\cos(\pi/6)+1}{2}}}{2}} = \sqrt{\frac{1+\sqrt{\frac{\sqrt{3}/2+1}{2}}}{2}} = \sqrt{\frac{1+\sqrt{\frac{\sqrt{3}+2}{4}}}{2}}$

(since $\pi/6, \pi/12, \pi/24 \in$ QI)

$= \sqrt{\frac{1+\sqrt{\sqrt{3}+2}/2}{2}} = \sqrt{\frac{2+\sqrt{\sqrt{3}+2}}{4}}$

$= \frac{\sqrt{2+\sqrt{\sqrt{3}+2}}}{2} = \frac{\sqrt{2+\sqrt{2+\sqrt{3}}}}{2}$

(b) $\sin 22^\circ \cos 23^\circ + \cos 22^\circ \sin 23^\circ$

$= \sin(22^\circ + 23^\circ)$

$= \sin(45^\circ)$

$= \sqrt{2}/2$

3. (a) Use the pythagorean identity and the angle sum formula for cosine to verify the double angle formulas for cosine.

$$\cos(2x) = \cos(x+x)$$
$$= \cos x \cos x - \sin x \sin x$$
$$= \cos^2 x - \sin^2 x$$
$$= (1-\sin^2 x) - \sin^2 x$$
$$= 1 - 2\sin^2 x$$
$$= \cos^2 x - (1-\cos^2 x) = 2\cos^2 x - 1$$

(b) Use the double angle formula for cosine to verify the power lowering formula for cosine.

$$\frac{1+\cos(2x)}{2} = \cos^2 x$$
$$\frac{\cancel{1} + 2\cos^2 x - \cancel{1}}{2} = \cos^2 x$$
$$\frac{\cancel{2}\cos^2 x}{\cancel{2}} = \cos^2 x$$
$$\cos^2 x = \cos^2 x$$

4. Solve for x:

$$\tan(4x) = \tan(x)$$

$$\tan(2\cdot 2x) = \tan(x)$$

$\therefore$ double angle for tangent

$$\frac{2\tan(2x)}{1-\tan^2(2x)} = \tan(x)$$

$\therefore$ cross-mult

$$2\tan(2x) = \tan(x)(1-\tan^2(2x))$$

$\therefore$ double angle for tangent (once on each side)

$$\frac{4\tan x}{1-\tan^2 x} = \tan x\left(1-\left(\frac{2\tan x}{1-\tan^2 x}\right)^2\right)$$

$\therefore$ mult. both sides w/ $(1-\tan^2 x)^2$

$$4\tan x(1-\tan^2 x) = \tan x\left((1-\tan^2 x)^2 - (2\tan x)^2\right)$$

$\therefore$ expand

$$4\tan x - 4\tan^3 x = \tan x(\tan^4 x - 2\tan^2 x + 1 - 4\tan^2 x)$$
$$4\tan x - 4\tan^3 x = \tan^5 x - 6\tan^3 x + \tan x$$

$$0 = \tan^5 x - 2\tan^3 x - 3\tan x$$

$\therefore$ Subs $u = \tan x$

$$0 = u^5 - 2u^3 - 3u$$
$$= u(u^4 - 2u^2 - 3) = u(u^2-3)(u^2+1)$$

$u = 0, \pm\sqrt{3}$

$x = 0 + \pi k$, $\pm\pi/3 + \pi k$, $k \in \mathbb{Z}$.

$x = \pi k/3, k \in \mathbb{Z}$