Exam III covers 10.2, 9.1-9.3, and 9.6. You should know/understand all of the following:

- Know the differentiation rules for logarithmic and exponential functions:

$$\begin{aligned}
\frac{d}{dx} \ln x &= \frac{1}{x} \\
\frac{d}{dx} \log_a x &= \frac{1}{x \ln a} \\
\frac{d}{dx} \ln g(x) &= \frac{g'(x)}{g(x)} \\
\frac{d}{dx} \log_a g(x) &= \frac{g'(x)}{g(x) \ln a} \\
\frac{d}{dx} e^x &= e^x \\
\frac{d}{dx} a^x &= (\ln a)\, a^x \\
\frac{d}{dx} e^{g(x)} &= g'(x)\, e^{g(x)} \\
\frac{d}{dx} a^{g(x)} &= (\ln a)\, g'(x)\, a^{g(x)} \\
\frac{d}{dx} e^{kx} &= k e^{kx}
\end{aligned}$$

- Graph a function $y = f(x)$ entirely by hand. This includes the following steps:
  - Find the x-intercepts by setting $f(x) = 0$ and solving for x
  - Find the y-intercept by evaluating $f(0)$
  - Find the horizontal asymptotes if the function is a rational function. For a rational function $f(x) = \frac{a_n x^n + \cdots + a_0}{b_m x^m + \cdots + b_0}$, use the following rule:
    (i) If $n < m$, the horizontal asymptote is $y = 0$
    (ii) If $n = m$, the horizontal asymptote is $\frac{a_n}{b_m}$
    (iii) If $n > m$, there is no horizontal asymptote
  - Find the vertical asymptotes (for a rational function, the vertical asymptotes are the zeros of the denominator, assuming no zero in the denominator also occurs in the numerator).
  - Find the critical numbers by setting $f'(x) = 0$ and finding where $f'(x)$ is undefined.
  - Determine which critical numbers are local minima, which are local maxima, and which are neither using one or both of the following tests:
    (i) The first derivative test: plug test values chosen from between each pair of adjacent critical numbers into the first derivative. To test a critical number $x = a$:
      * If the sign of the first derivative changes from positive to negative at $x = a$, then $x = a$ is a local maximum.
      * If the sign changes from negative to positive, then $x = a$ is a local minimum.
      * If the sign does not change, then $x = a$ is neither a maximum nor a minimum.

      So if your critical numbers are $x = -2, 3$, and 7, you would evaluate $f'(x)$ at $-5, 0, 5$, and 10. If the results were $f'(-5) = -10$, $f'(0) = -3$, $f'(5) = 6$, and $f'(1) = 11$, you would conclude that a minimum occurs at $x = 3$, and that the critical numbers at $x = -2, 7$ are neither maxima nor minima.

    (ii) The second derivative test: plug critical numbers into the second derivative. To test the critical number $x = a$:
      * If $f''(a) > 0$, then $x = a$ is a local minimum.
      * If $f''(a) < 0$, then $x = a$ is a local maximum.
      * If $f''(a) = 0$, then the second derivative test gives no information and you must use the first derivative test instead.

So if your critical numbers are $x = -2, 3$, and 7, you would evaluate $f''(-2), f''(3)$, and $f''(7)$. If $f''(-2) = 0, f''(3) = 10$, and $f''(7) = 0$, you could conclude that $x = 3$ is a local minimum, but you could conclude nothing about $x = -2$ or $x = 7$ without falling back to the first derivative test.

- Find all intervals of increase/decrease, i.e. make a sign chart for the first derivative.
- Find all possible inflection points by setting $f''(x) = 0$ and finding where $f''(x)$ is undefined.
- Determine which of the possible inflection points are actual inflection points by checking whether the sign of the second derivative changes there. For example, with possible inflection points at $x = \pm 2$, you could evaluate $f''(-4), f''(0)$, and $f''(4)$. If the results were $f''(-4) = 10, f''(0) = 15$, and $f''(4) = -5$, you would conclude that $x = 2$ is an inflection point but $x = -2$ is not.
- Determine the intervals of concavity, i.e. make a sign chart for the second derivative.
- Sketch a graph that combines all the other information you found in the previous steps.

- Know how to solve optimization problems. This involves the following general steps:

1. Draw a diagram and label any unknowns/constants given in the statement of the problem.
2. Write down all relevant equations you can think of involving those unknowns.
3. Determine which equation needs to be optimized, and whether it needs to be maximized or minimized (usually this will be clear from the wording of the problem).
4. If the equation from step 3 has multiple variables, you will need to eliminate all but one of the variables as follows:
   (a) Solve for the other variable(s) in your other equation(s)
   (b) Substitute your expression(s) for the other variable(s) into the equation from step 3.
5. Find all critical numbers of this equation.
6. Find the domain of the equation.
7. Determine which optimization procedure to use. If you have a closed interval for the domain, you can use the closed interval method:
   (a) Find the value of the function at all critical numbers and at each endpoint of your interval.
   (b) The largest of these values is the global maximum, the smallest is the global minimum.

   If you have only a single critical point $x = a$, you can use the second derivative test for global extrema:
   (a) If $f''(a) < 0$, then $x = a$ is a local maximum, the only critical number, and hence a global maximum.
   (b) If $f''(a) > 0$, then $x = a$ is a local minimum, the only critical number, and hence a global minimum.

   If your domain is not a closed interval and you have multiple critical numbers, neither of these methods apply (this will not happen on the exam – if it does, you have made a mistake somewhere).
8. Go back to the original problem and determine which of the variables you have been asked to find, and solve for this variable if needed. Include units with your answer.

- Know how to differentiate implicitly. Basically, this involves breaking down the equation using the product/quotient/chain rule until each subexpression involves only x or only y, and ensuring that you always multiply by y' when differentiating a subexpression involving only y.

- Know how to use implicit differentiation to find the equation of a tangent line to a curve defined implicitly:

  1. Find the y-value(s) corresponding to the given x-value.
  2. Differentiate implicitly to get an equation for y' in the variables x and y
  3. Substitute your (x, y)-values into the equation for y' to get m.
  4. Use the point-slope form of the equation of the tangent line.