Final Exam

Name (35 pts): _____

All problems are worth 50 points. Show all your work (excluding arithmetic) for full credit.

1. For each of the following limits, find the limit algebraically or show that the limit does not exist:

   (a) $\lim_{x\to -3^-} \frac{\sqrt{x^2+6x+9}}{x+3}$

   (b) $\lim_{x\to -3^+} \frac{\sqrt{x^2+6x+9}}{x+3}$

   (c) $\lim_{x\to -3} \frac{\sqrt{x^2+6x+9}}{x+3}$

   (d) $\lim_{x\to 0^+} \ln x$

   (e) $\lim_{x\to 0} \frac{e^{2x}-2e^x+1}{e^x-1}$

2. Find the following derivatives:

(a) $\frac{\mathrm{d}}{\mathrm{d}x}\left[\sqrt{x}+\sqrt{e}+\sqrt{e^x}+e^{\sqrt{x}}\right]$

(b) $\frac{\mathrm{d}}{\mathrm{d}x}\left[\frac{\ln(x)}{\ln(x)+1}\right]$

(c) $\frac{\mathrm{d}}{\mathrm{d}y}\left[\frac{\ln(x+1)}{\ln(x)}\right]$

(d) $\frac{\mathrm{d}}{\mathrm{d}x}\left[(3x+4)^7\sqrt{5x+6}\right]$

(e) $\frac{\mathrm{d}}{\mathrm{d}x}e^{e^{e^x}}$

3. Find the following indefinite integrals:

(a) $\int \frac{x^2-x-12}{x^2+3x} \mathrm{d}x$

(b) $\int \left[\sqrt{x} - \frac{1}{\sqrt{x}}\right] \mathrm{d}x$

(c) $\int \left(e^{(x/2)} - 2\right)^2 \mathrm{d}x$

(d) $\int \frac{xy}{z} \mathrm{d}x$

(e) $\int \frac{xy}{z} \mathrm{d}z$

4. Use the definition of the derivative to find

$$\frac{\mathrm{d}}{\mathrm{d}x}\frac{1}{\sqrt{5x-1}}$$

5. Suppose a flock of geese begins migrating south for the winter on October 31. Its velocity after t days is given in miles/day by $v(t) = 400 - \frac{200}{(t+1)^2}$. Find the flock's acceleration at the end of the trip if it has 2000 miles to travel.

6. Find

$$\int \left[\frac{1}{x \ln (x^2 + 2x)} + \frac{1}{(x+2) \ln (x^2 + 2x)} \right] \mathrm{d}x$$

7. Suppose a cylindrical can is to be formed from sheet metal by punching the top and bottom out of square metal pieces and then wrapping a flat rectangular piece around to form the sides. Determine the dimensions that minimize the amount of metal used, assuming the remnants of the square pieces will be discarded as scrap and the can must have a capacity of 500 ml.