Exam 3

Name (35 pts): _____

All problems are worth 50 points. Show all your work (excluding arithmetic) for full credit.

1. For each of the following limits, find the limit algebraically or show that the limit does not exist:

   (a) $\lim_{x\to 3} \frac{x^4-18x^2+81}{x^2-9}$

   (b) $\lim_{x\to 5^+} \frac{|x-5|}{x-5}$

   (c) $\lim_{x\to 5} \frac{|x-5|}{x-5}$

   (d) $\lim_{x\to 4} \frac{9x}{x^2+2}$

   (e) $\lim_{x\to 0} \frac{1}{x^2}$

2. Find the following derivatives:

(a) $\frac{\mathrm{d}}{\mathrm{d}x} \ln\left(\sqrt{x} + x^2\right)$

(b) $\frac{\mathrm{d}}{\mathrm{d}x} \frac{e^x}{x^e}$

(c) $\frac{\mathrm{d}}{\mathrm{d}x} x^2 \ln x$

(d) $\frac{\mathrm{d}}{\mathrm{d}x} \frac{\left(x^3+3\right)^2}{x}$

(e) $\frac{\mathrm{d}}{\mathrm{d}x} \frac{e^{\sqrt{x}}}{\sqrt{x+e^x}}$

3. Find the following indefinite integrals:

(a) $\int \sqrt{e^x} \mathrm{d}x$

(b) $\int \frac{x^2-25}{x+5} \mathrm{d}x$

(c) $\int \frac{3x^2+4}{3x} \mathrm{d}x$

(d) $\int 5 \mathrm{d}x$

(e) $\int 5 \mathrm{d}r$

4. Suppose the ice sheet north of Narsaq, Greenland is thinning at a rate given (in m/yr) by $r(t) = 0.05 + 0.06\sqrt{t}$, where t is the number of years since 1900. Determine how thick the ice sheet was in 1975 if it was 500 m thick in 2005.

5. Suppose the volume of a vase t seconds after a glass smith begins blowing in air is given (in in^3) by $V(t) = 150 - \frac{150}{t+1}$. Use the definition of the derivative to find a formula for $V'(t)$. Use your formula to determine the value of $V'(5)$. Give units and interpret your answer.

6. Suppose a box with a square base is to be constructed from steel with a capacity of 1000 in^3. Find the dimensions of the box that minimize the amount of metal used in construction.

7. Graph $e^{(-x^2)}$ by hand. Show all features that can be determined using the function's derivatives.