

Limit Laws

Suppose $\lim_{x \rightarrow a} f(x)$ and $\lim_{x \rightarrow a} g(x)$ exist and are finite. (Note that a can be either a finite number or $\pm\infty$). Then we have the following rules:

1. $\lim_{x \rightarrow a} [f(x) \pm g(x)] = [\lim_{x \rightarrow a} f(x)] \pm [\lim_{x \rightarrow a} g(x)]$
2. $\lim_{x \rightarrow a} [f(x)g(x)] = [\lim_{x \rightarrow a} f(x)] [\lim_{x \rightarrow a} g(x)]$
3. $\lim_{x \rightarrow a} [cf(x)] = c[\lim_{x \rightarrow a} f(x)]$
4. $\lim_{x \rightarrow a} c = c$
5. If $[\lim_{x \rightarrow a} g(x)] \neq 0$, then $\lim_{x \rightarrow a} \left[\frac{f(x)}{g(x)} \right] = \frac{\lim_{x \rightarrow a} f(x)}{\lim_{x \rightarrow a} g(x)}$
6. If $f(x) \leq g(x)$, then $[\lim_{x \rightarrow a} f(x)] \leq [\lim_{x \rightarrow a} g(x)]$
7. If $f(x) \leq g(x) \leq h(x)$ and $\lim_{x \rightarrow a} f(x) = \lim_{x \rightarrow a} h(x) = L$, then $\lim_{x \rightarrow a} g(x) = L$

The remaining rules can be used when one of $\lim_{x \rightarrow a} f(x)$ or $\lim_{x \rightarrow a} g(x)$ is infinite:

8. If $[\lim_{x \rightarrow a} f(x)] \neq 0$ and $[\lim_{x \rightarrow a} g(x)] = 0$, then $\lim_{x \rightarrow a} \left[\frac{f(x)}{g(x)} \right] = \pm\infty$, depending on whether $\frac{f(x)}{g(x)}$ is positive or negative for large values of x .
9. If $[\lim_{x \rightarrow a} f(x)] = \pm\infty$ and $[\lim_{x \rightarrow a} g(x)] \neq \pm\infty$, then $\lim_{x \rightarrow a} \left[\frac{f(x)}{g(x)} \right] = \pm\infty$, depending on whether $\frac{f(x)}{g(x)}$ is positive or negative for large values of x .
10. If $[\lim_{x \rightarrow a} f(x)] \neq \pm\infty$ and $[\lim_{x \rightarrow a} g(x)] = \pm\infty$, then $\lim_{x \rightarrow a} \left[\frac{f(x)}{g(x)} \right] = 0$