Math 567, fall 2008- Problems for lecture 4. (due 9/11, source for 1: F.Warner, p.51)

1. Let $f : M \to N$ be an immersion, X a vector field on M. Suppose that $df(p)[X_p] = df(q)[X_q]$ whenever $f(p) = f(q)$. Is there a smooth vector field Y on N which is f-related to X?

2. Give a proof using local charts of the fact that if $f : M \to N$ is an immersion, X is f-related to Y and X_1 is f-related to Y_1 (X, X_1 vector fields on M; Y, Y_1 vector fields on N), then $[X, X_1]_M$ is f-related to $[Y, Y^1]_N$. What is the smallest class of differentiability you need for the proof to work?