

# Linear Algebra in the grand scheme of things

*Mathematics* arises from the natural human impulse to impose logical and quantitative structures on the natural and social worlds and on designed objects. This is achieved by means of *models*, simplified descriptions that capture the essential features of a situation, and make possible:

# Linear Algebra in the grand scheme of things

*Mathematics* arises from the natural human impulse to impose logical and quantitative structures on the natural and social worlds and on designed objects. This is achieved by means of *models*, simplified descriptions that capture the essential features of a situation, and make possible:

*quantitative prediction*

# Linear Algebra in the grand scheme of things

*Mathematics* arises from the natural human impulse to impose logical and quantitative structures on the natural and social worlds and on designed objects. This is achieved by means of *models*, simplified descriptions that capture the essential features of a situation, and make possible:

*quantitative prediction*

*optimal design*

# Linear Algebra in the grand scheme of things

*Mathematics* arises from the natural human impulse to impose logical and quantitative structures on the natural and social worlds and on designed objects. This is achieved by means of *models*, simplified descriptions that capture the essential features of a situation, and make possible:

*quantitative prediction*

*optimal design*

Mathematical reasoning falls into four distinct ‘modes’, corresponding to the four different ‘trunks’ of the subject:

## The Four Modes of Mathematics

Algebra (algorithms)

Analysis (approximation)

Geometry (pictures)

Probability (randomness)

# Some branches of mathematics

## Basic Mathematics

Algebraic geometry

Number theory

Topology

Differential Geometry

Dynamical systems

Stochastic processes

# Some branches of mathematics

## Basic Mathematics

Algebraic geometry

Number theory

Topology

Differential Geometry

Dynamical systems

Stochastic processes

## Applied Mathematics

Partial Differential Equations

Fluid mechanics

Graph theory

Coding and information theory

Mathematical statistics

General Relativity

## The role of linearity

In any modeling situation (in *any* of the fields above), the first instances of the model to be understood (and often the only ones which are *exactly computable*) are linear ones:

input twice as strong implies twice the effect

## The role of linearity

In any modeling situation (in *any* of the fields above), the first instances of the model to be understood (and often the only ones which are *exactly computable*) are linear ones:

input twice as strong implies twice the effect

If input(1) yields effect(1) and input(2) yields effect(2), then:  
input=input(1)+input(2) yields: effect=effect(1)+effect(2)

# The role of linearity

In any modeling situation (in *any* of the fields above), the first instances of the model to be understood (and often the only ones which are *exactly computable*) are linear ones:

input twice as strong implies twice the effect

If input(1) yields effect(1) and input(2) yields effect(2), then:  
input=input(1)+input(2) yields: effect=effect(1)+effect(2)

There are many areas of mathematics (including many of engineering interest) in which the *only* classes of models mathematically understood at present are linear ones; more general ones are reduced to the linear case by approximation (*linearization*), which often works only in ‘small input’ or ‘perturbation from exact’ situations.