Problems on Laplace transforms.

1. Find an inverse Laplace transform $f(t)$, given $F(s)$;

$$(i) F(s) = \frac{s^2 - 5}{s^3 + 4s^2 + 3s} \quad (ii) F(s) = \frac{3s}{(s+1)^4} \quad (iii) F(s) = \frac{s}{s^2 - 3s + 12}.$$

2. Solve the initial-value problems using Laplace transforms:

$$(i) y'' + y = t \sin t, \quad y(0) = 1, y'(0) = 2;$$

$$(ii) y'' - 2y' + y = te^t, \quad y(0) = y'(0) = 0;$$

$$(iii) y'' + y' + y = 1 + e^{-t}; \quad y(0) = 3, y'(0) = -5.$$

3.Find the transfer function $H(s)$ and impulse response function $h(t)$, and give a formula for the solution of the problem.

$$(i) y'' - y' - 6y = g(t); \quad y(0) = 1, y'(0) = 8.$$

$$(ii) y'' + 2y' + 5y = g(t); \quad \text{(steady-state solution)}$$

Remark: The 'transfer function' $H(s)$ is the Laplace transform of the solution of the homogeneous equation with initial conditions $y(0) = 0, y'(0) = 1$; the 'impulse response function' is the inverse Laplace transform of $H(s)$.

4. Find an inverse Laplace transform (and sketch its graph):

$$(i) \frac{e^{-5s}}{s^2} \qquad (ii) \frac{e^{-4s}}{s^2 + 9}$$

5. Use the convolution theorem to find(explicitly) an inverse Laplace transform:

$$(i) \frac{s}{(s^2+1)^2} \quad (ii) \frac{s+1}{(s^2+4)^2}$$