

**Math 231 spring 2009- Exam 1, 2/5/09** No credit for answers without justification. Closed books, closed notes. Calculators OK. Time given: 75 minutes. **(Five problems, 20 pts each).**

1. Consider the linear initial-value problems for  $y(x)$ .

(i)  $y' + 2y = -4$ ,  $y(0) = 1$ . Find the solution, sketch its graph.

(ii)  $(1-x^2)y' + \sqrt{2x+3}y = \ln|x|$ ,  $y(-1/2) = 2$ . Find the domain of the solution.

2. Show that the following first-order equation is not exact, but admits  $y^{-3}$  as an integrating factor. Find an implicit equation for the integral curve through the point  $(1, 1)$ .

$$y(2x + y^3)dx - x(2x - y^3)dy = 0.$$

3. A tank initially contains 200 l of fresh water. It receives brine of an unknown salt concentration at the rate of 2 l/min. The mixture flows out at the same rate. At the end of 120 min, 140 kg of salt are in the tank. Find the salt concentration of the entering brine (in kg/l).

4. Solve the following non-linear initial-value problems for  $y(x)$  (explicit solution). Include the domain of the solution (recall this must be an *interval*, possibly all of  $\mathbb{R}$ ).

(i)  $y' + y = xy^2$ ,  $y(0) = 1$ .

(ii)  $(xy - y^2)dx - x^2dy = 0$   $y(1) = 1$ . (homogeneous type)

5. For the autonomous first-order equation given below: (i) find and classify the equilibria (constant solutions), as stable or unstable. (ii) Draw a diagram of all solutions, showing two solutions in each range determined by the equilibria. (iii) Indicate in your diagram which solutions have domain  $\mathbb{R}$ , and which are defined in an interval; in the latter case, show also the vertical asymptotes for typical solutions.

$$y' = -y(y - 2)(y + 1), \quad y = y(x).$$