Math 231 fall 2008- Exam 1, 9/25/08 No credit for answers without justification. Calculators OK. Time given: 75 min.

1. A tank initially contains 200 l of fresh water. It receives a saline solution of unknown concentration, at the rate of 2 l/min; the mixture flows out at the same rate. At the end of 100 min, 200 kg of salt are in the tank. (i) Find the concentration of salt in the entering solution (in kg/l). (ii)Sketch the graph of concentration as a function of time.

2. For the autonomous equation given: (i) find and classify the equilibria (as stable or unstable); (ii) sketch the diagram of all solutions (including at least two curves in each region defined by the equilibria; indicate vertical asymptotes where needed); (iii) identify the initial conditions (at $t = 0$) corresponding to solutions defined for all $t \in \mathbb{R}$, and those with solutions defined only on an interval $(-\infty, T_*)$ or (T_*, ∞).

$$y' = (y^2 + 1)(y^2 - 1), \quad y = y(t).$$

3. (i) Find the *domain* of the solution of the initial-value problem. (This can be done without solving the equation; justify your answer):

$$y' + \frac{2x}{x^2 - 1} y = \frac{1}{x(x^2 - 1)}, \quad y = y(x), \quad y(1/2) = 1.$$

(ii) Find the *general solution* of the equation in (i).

4. Consider the equation:

$$\omega = (\frac{2xy - 1}{y})dx + (\frac{y + x}{y^2})dy = 0.$$

(i) Find a primitive $E(x, y)$ for ω, including its domain (which should be a*connected* and *simply connected* region $U \subset \mathbb{R}^2$.)

(ii) For which points (x_0, y_0) of the plane is there guaranteed to be exactly one solution curve of $\omega = 0$ going through (x_0, y_0)?

5. For the following equation of "homogeneous type": (i)find the solution of the initial-value problem in implicit form, then (ii) solve for y as an explicit function of x and include the largest open interval (containing $x_0 = 1$) in which $y(x)$ is defined.

$$(xy - y^2)dx - x^2 dy = 0, \quad y = y(x) \quad y(1) = 1$$

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