

MATH 241- EXAM 4- November 29, 2006. **NAME:**

1. (*homework-Stewart*) A fluid has density 800 kg/m^3 , and flows with velocity vector field $\vec{v} = (z, y^2, x^2)$ (components in meters per second). Find the time rate of change of the total mass of fluid inside the region bounded by the cylinder $x^2 + y^2 = 4, 0 \leq z \leq 1$, in kilograms per second. (This is the same as the rate of flow outward through the boundary of the region).

2. (*homework-Stewart*) Use Gauss's law to find the charge contained in the hemisphere $x^2 + y^2 + z^2 \leq 1, z \geq 0$, if the electric field on the boundary of the region is given by $\vec{E} = (x, y, 2z)$.

3. (*handout, probl.2*) Use Stokes' theorem to show that, if C is the intersection of the sphere $x^2 + y^2 + z^2 = a^2$ and the plane $x + y + z = 0$, we have:

$$\int_C ydx + zdy + xdz = \pi a^2 \sqrt{3}.$$

Specify how C must be oriented.

4. (*homework-Stewart*) Let C be the curve of intersection of the surfaces $z = y^2 - x^2$ and $x^2 + y^2 = 1$, oriented counterclockwise as seen from above. Let S be *any* oriented surface with boundary C , and consider the vector field:

$$\vec{F} = (ax^3 - 3xz^2, x^2y + by^3, cz^3).$$

(i) Explain in detail why there is a condition on \vec{F} which guarantees that the value of $\int \int_S \vec{F} \cdot \vec{N} dS$ is independent of the surface S spanning C . (*Hint:* The argument uses either Stokes' theorem or the divergence theorem.)

(ii) For the given \vec{F} , find the values of a, b, c for which \vec{F} satisfies the condition.

5. (*cp. hw problem 13.6.25*) Find the flux of the vector field $\vec{v} = (2y, x, -z)$ across the surface:

$$S = \{(x, y, z); z = x^2 - 2y^2, x^2 + y^2 \leq 9\}$$

(a hyperbolic paraboloid, given as a graph over a disk of radius 3), oriented by the *upward* unit normal.