MATH 142- EXAM 3- November 21, 2006.

**1.** Do the following improper integrals converge or diverge? Justify.

$$(a) \int_0^2 \frac{dx}{\sqrt{4-x^2}}.$$

(Use $4-x^2 = (2-x)(2+x) \sim 4(2-x)$ as $x \to 2$.)

$$(b) \int_0^\infty \frac{\sin^2 x}{\sqrt{1+x^3}} dx.$$

**2.**(i) Find the limit of the sequence:

$$\sqrt{2}, \sqrt{2\sqrt{2}}, \sqrt{2\sqrt{2\sqrt{2}}}, \ldots$$

*assuming* the limit exists. (Note that the sequence is given recursively by $a_{n+1} = \sqrt{2}\sqrt{a_n}$.)

(ii)Explain why $a_n < 2$ implies $a_{n+1} < 2$.

**3.** Express the rational number below as the ratio of two integers:

$$0.\overline{73} = 0.737373\ldots$$

**4.** For each of the two series below, answer: how many terms of the series would you need to add to find its sum to within 0.01?

$$(i) \sum_{n=1}^\infty \frac{1}{n^4} \quad (ii) \sum_{n=1}^\infty \frac{(-1)^n n^2}{10^n}.$$

**5.** Decide convergence/divergence, using an appropriate test.

$$(i) \sum_{n=1}^\infty \frac{(-1)^n}{n^{1/7}} \quad (ii) \sum_{n=1}^\infty \frac{2^n}{n!} \quad (iii) \sum_{n=1}^\infty \frac{\sqrt{n}}{1+n^2} \quad (iv) \sum_{n=1}^\infty \frac{1+\sin(\sqrt{n})}{n^2}.$$

**6.** Express the function below as a power series at 0, and give the radius of convergence.

$$f(x) = \frac{x^2}{x^4+9}.$$