Math 142- Exam 2B (optional)- November 3, 2006

1. Find the total **arc length** of the loop of the curve given by the parametric equations (see diagram):

$$x(t) = 3t - t^3, \quad y(t) = 3t^2$$

(finding the correct limits of integration is already half the problem.)

2.Find the **area** A_n enclosed by one turn of the logarithmic spiral with polar coordinate equation $r = e^\theta$, defined by the limits $2n\pi \leq \theta \leq (2n+2)\pi$. Here $n \geq 0$ is an arbitrary integer, and the answer of course depends on n.

3. Find the **volume** of the solid constructed in the following way: the base is the region $D = \{(x, y); y^2 \leq x \leq 1\}$, and cross-sections perpendicular to the x axis are equilateral triangles.

4. Find the **centroid** of the region in $\mathbb{R}^2$:

$$Q = \{(x, y); x^2 + y^2 \leq 4, y \geq 0 \text{ and } (|x| \geq 1, \text{ or } y \geq 1)\}$$

(diagram given)

5. Given a point $P = (x_0, y_0)$ on the $y > 0$ branch of the hyperbola:

$$y^2 - x^2 = 1,$$

(i)[5] write down the equation of the **tangent line** to the hyperbola at (x_0, y_0).
(ii)[5] **Show** that if Q is the foot of the perpendicular drawn from P_0 to the axis, T is the point where the tangent at P intersects the axis and O is the origin, we have:

$$|OT| \cdot |OQ| = 1.$$

6. From observation, it is known that the period of revolution of *Mercury* around the sun is 0.241 earth years. Use **Kepler's third law** to estimate the distance in AU from *Mercury* to *Earth*(*not from Mercury to the Sun!*), at the point of closest approach between them.