

Math 142- Exam 2- October 23, 2006
(Diagrams were provided for each problem)

1. Find the total arc length of the *astroid*, given by the parametric equations ($a > 0$ arbitrary):

$$x(t) = a \cos^3 t, \quad y(t) = a \sin^3 t, \quad t \in [0, 2\pi].$$

2.(i)[5] Find the values of θ for which $(8 + 8 \sin \theta) \sin \theta = 4$.

(ii)[5] Set up an integral to compute the area of the region inside both the cardioid $r = 8 + 8 \sin \theta$ and the horizontal strip $4 \leq y \leq 18$. Explain how the limits of integration are found

3. Find the volume of the solid constructed in the following way: the base is the region $D = \{(x, y); x^2 \leq y \leq 1\}$, and cross-sections perpendicular to the y axis are squares.

4. Find the centroid of the region in \mathbb{R}^2 :

$$Q = \{(x, y); 1 \leq x^2 + y^2 \leq 4, x \geq 0, y \geq 0\}$$

5. Given a point P on a parabola with vertex V , let T be the point where the tangent to the parabola at P intersects its axis, and Q the foot of the perpendicular dropped from P to the axis. Show that $|QV| = |TV|$. (*Hint*: set up a convenient cartesian coordinate system (x, y) .)

6. From parallax measurements, it is found that the distance between earth and Venus at the point of closest approach between them is about 0.3 AU. Use Kepler's third law to estimate the period of revolution of Venus around the Sun, measured in earth years. (Pay attention to the diagram given.)