1. (ii) f_{even} is continuous with piecewise continuous derivative, so $s_N \to f$ uniformly in $[0, \pi]$.

(iii) The pointwise limit $g(x)$ of $\sigma_N(x)$ equals 1 on $(\pi/2, \pi)$, -1 on $(0, \pi/2)$ and 0 at $x = 0, \pi/2, \pi$. $(f')_{odd}$ is not continuous on $\mathbb{R}$, so the convergence $\sigma_N \to g$ is **not** uniform on $[0, \pi]$.

2.(i) *Maximum principle:* If $u(x, t)$ is a solution of the heat equation on $\bar{D} \times [0, T]$, the maximum of u in this region is attained either at $t = 0$ or on the boundary of D.

(ii) The maximum value of $|u(x, t)|$ at $t = 0$ is 1, as is the maximum on the boundary, for any $t \geq 0$. By the maximum principle, we may take $M = 1$.

3. $u(x, y, t) = \frac{y}{\pi} + e^{-5t} \cos x \sin 2y$.

4. (i) Since the initial condition g is radial, only $J_{1/2}$ appears in the formal solution:

$$u(r, t) \sim \sum_{j=1}^{\infty} c_j \sin(z_j t) \frac{1}{\sqrt{r}} J_{1/2}(z_j r),$$

where z_j is the jth. positive zero of $J_{1/2}$ and the coefficients are determined by the expansion:

$$g(r) \sim \sum_{j=1}^{\infty} z_j c_j \frac{1}{\sqrt{r}} J_{1/2}(z_j r), \quad r \in [0, 1].$$

(ii) From the information in the handout we see that $z_1 = \pi$ and $g(r) = (const.) \frac{1}{\sqrt{r}} J_{1/2}(z_1 r)$, so only $j = 1$ occurs in the solution, and we have:

$$u(r, t) = \frac{\sin \pi r}{\pi r} \sin \pi t,$$

for $r \neq 0$ ($u(0, t) = \sin \pi t$.)

5. (i) The most general solution is:

$$u(r, \theta) = a + br^2 \sin 2\theta + cr^{-2} \sin 2\theta, \quad b - c = \frac{1}{2}.$$

(ii) If we require: $u(r, \theta) \to 0$ as $r \to \infty$, the coefficients a and b must vanish, and the solution is unique, $u(r, \theta) = (-1/2) r^{-2} \sin 2\theta$.

Note that if we merely assume u is *bounded* for $r > 1$, the constant a is still undetermined.

6. (i) $G_D(x; y)$ is harmonic in $D - \{y\}$, as a function of x, and equals zero for $x \in \partial D$, $y \in D$; and $G_D(x; y) - G(x; y)$ is a smooth harmonic function of $x \in D$ (for any fixed $y \in D$), where $G(x; y)$ is the 'whole-space Green's function'.

(ii) Follows from the expressions in the handout and the fact $|x - \bar{y}| = |\bar{x} - y|$, easily checked geometrically.

(iii) Changing to polar coordinates (r, θ):

$$\frac{x_2}{x_1^2 + x_2^2} = \frac{1}{r} \sin \theta,$$

which is harmonic.