MATH 400- HISTORY OF MATHEMATICS- SUMMER 2006

Instructor: Dr. Alex Freire (Ayres 207 A, 974-4313, freire@math.utk.edu, www.math.utk.edu/~freire Office hours: right after class, or appointment by e-mail MTWRF, June 5 to July 7, Ayres 309A, 9:45- 11:15

Grading: (1) Two mini-biographies (details to be announced); (2) three written in-class exams. *Remark:* this is a `writing intensive' course, and also `reading intensive'.

Prerequisite: calculus; also: must enjoy learning mathematics (this will *not* be purely a survey course- learning a little analysis will be an integral part of it). *Remark:* let me know about your background in mathematics and current educational goal.

Main theme: *the development of Analysis, from Newton to Lebesgue.*

Secondary topics: *mathematics in Ancient Greece/ Analysis before Newton and Leibniz/ early history of the Calculus of Variations/ elliptic integrals and elliptic functions/ the Prime Number Theorem/ transcendence theorems*

MATHEMATICS IN ANCIENT GREECE

Pythagoras and his school/ Plato and Eudoxus/ Euclid's *Elements/* Archimedes/ Apollonius

Sources: Struik, Ch. 3 and *handout.*

ANALYSIS BEFORE NEWTON AND LEIBNIZ

Rene' Descartes (1596-1650): *the equation of a curve/* Bonaventura Cavalieri (1598-1647): *integration of higher parabolas/* Pierre de Fermat (1601-1665): *maxima and minima/ Fermat's Principle of Least Time*

Sources: Struik, Ch. 6 and *handout*

HISTORY OF ANALYSIS, I: NEWTON to EULER and LAGRANGE

Source: The Calculus Gallery, W. Dunham (Princeton U.P., 2005)

Chapter 1: IsaacNewton (1642-1727)

Generalized binomial expansion (1665)/quadrature rules/derivation of the sine series

Newton's *Principia* (Mathematical Principles of Natural Philosophy, 1687)/*motion in a resisting medium*

Chapter 2: Gottfried Wilhelm Leibniz (1646-1727)

`Transmutation theorem' /Leibniz series for π/4

Chapter 3: Jakob (1654-1705) and Johann (1667-1748) Bernouilli

Divergence of the harmonic series (Jakob) /sums of `figurate series' (Jakob)/ integral of x to the power x (Johann)

The Brachystochrone Problem (Johann; also Newton and Leibniz)

Chapter 4: Leonhard Euler (1707-1783)

Integral of sin(lnx)/ln(x)/ estimation of π/sums of infinite series/the Gamma function/the Euler product

Chapter 5: First Interlude
Joseph-Louis Lagrange (1736-1813)- *birth of the Calculus of Variations (also Euler)*

Carl-Friedrich Gauss (1777-1855)
Constructible polygons/ fundamental theory of algebra

The birth of elliptic function theory: addition formulas for elliptic functions
Euler and Adrien-Marie Legendre (1752-1833)
Niels-Henrik Abel (1802-1829) and Carl Gustav Jacob Jacobi (1804-1851)

Chapter 6: Augustin-Louis Cauchy (1789-1857)
Intermediate value theorem, mean value theorem and the fundamental theorem of calculus/ two convergence tests for series

BERNHARD RIEMANN and the PRIME NUMBER THEOREM

Source: *Prime Obsession, by John Derbyshire*

Ch.1: divergence of the harmonic series (`card trick')
Ch.2: Georg-Friedrich Bernhard Riemann (1826-1866) –early life, German states c. 1826
Ch.3: The Prime Number Theorem: motivation and statement
Ch.4: Gauss to 1806/ Gauss in Gőttingen/ Gauss and the PNT/ Legendre and the PNT/ Russia under Peter the Great/ Euler in St. Petersburg/ Euler in Berlin
Ch.5: The zeta function
Ch. 6: Statement of the Riemann Hypothesis/ Peter Gustav Lejeune Dirichlet (1805-1859)/ Dirichlet's Prime Number Theorem
Ch.7: Eratosthene's sieve/ Euler product formula (`golden key') /PNT in terms of *Li*
Ch.8: Riemann in Gőttingen/ Pafnuti Lvovich Chebyshev (1821-1894)/ Chebyshev's PNT/ Chebyshev bias/ Riemann's habilitation/ Richard Dedekind (1831-1916)/Riemann succeeds Gauss/ Riemann's 1859 paper
Ch.9: analytic extension of the zeta function

Ch.10: progression to a proof of the PNT (1896): politics in France/Charles Hermite (1822-1901) and Thomas Joannes Stieltjes (1856-1894)/ Charles de la Vallee Poussin (1866-1962) and Jacques Hadamard (1865-1963)

HISTORY of ANALYSIS, II: CAUCHY to WEIERSTRASS

Chapter 7: Georg Friedrich Bernhard Riemann (1826-1866)
Joseph Fourier (1768-1830): expansions in trigonometric series/ Dirichlet: convergence of trigonometric series, nowhere continuous function/ Riemann's Habilitationsschrift (1854): definition of Riemann integral, integrability condition/ integrable function with countably many discontinuities/rearrangement of series.

Chapter 8: Joseph Liouville (1809-1822)
Existence of transcendental numbers (also Hermite- transcendence of e)/Liouville's theorem and elliptic functions

Chapter 9: Karl Weierstrass (1815-1897)
Uniform continuity and uniform convergence/ continuous, nowhere differentiable function/ Weierstrass and elliptic functions/ the Lindemann-Weierstrass theorem (transcendence of π)

Chapter 10: Second Interlude

HISTORY OF ANALYSIS, III: CANTOR to LEBESGUE

Chapter 11: Georg Cantor (1845-1918)
Completeness and construction of the real numbers (also Dedekind) /uncountability of intervals and the existence of transcendental numbers

Chapter 12: Vito Volterra (1860-1940)
Sets of discontinuity

Chapter 13: Rene' Baire (1874-1932)
The Baire category theorem/ characterization of sets of discontinuity/ classification of discontinuous functions

Chapter 14: Henri Lebesgue (1875-1941)
Characterization of Riemann-integrable functions/ sets of measure zero/ measurable sets and measurable functions/ the Lebesgue integral

REFERENCES

1) Main sources

W. Dunham, *The Calculus Gallery: Masterpieces from Newton to Lebesgue* (Princeton 2005)

J. Derbyshire, *Prime Obsession: Bernhard Riemann and the Greatest Unsolved Problem in Mathematics* (Penguin/Plume, 2004) [Part I, chapters 1 to 10]

D. Struik , *A Concise History of Mathematics* (Dover 1987)

2) Secondary sources

W. Dunham, *Journey through Genius: the great theorems of mathematics* (Penguin 1991)

D. Struik (ed.), *A Source Book in Mathematics, 1200-1800* (Harvard 1969)

H. Goldstine, *A History of the Calculus of Variations from the 17th. Through the 19th. Century* (Springer-Verlag New York, 1980)

R.Laubenbacher, D.Pengelley, *Mathematical Expeditions: Chronicles by the Explorers* (Springer-Verlag New York, 1999)

Sir Thomas Heath, *A Manual of Greek Mathematics* (Dover 1963)

W. Dunham, *Euler: the Master of Us All* (MAA 1999)