ARCHIMEDES' `*On Floating Bodies*'

BOOK 1: basic principles of hydrostatics

Proposition 2: *The surface of any fluid at rest is the surface of a sphere whose center is the same as that of the earth.*

Proposition 3: *Of solids, those which, size for size, are of equal weight with a fluid will, if let down into the fluid, be immersed so that they do not project above the surface but do not sink lower.*

Proposition 6: *If a solid lighter than a fluid is forcibly immersed in it, the solid will be driven upwards by a force equal to the difference between its weight and the weight of the fluid displaced.*

Proposition 7: *A solid heavier than a fluid will, when placed in it, descend to the bottom of the fluid, and the solid will, when weighted in the fluid, be lighter than its true weight by the weight of the fluid displaced.*

Propositions 8/9: *If a solid in the form of a segment of a sphere, and of a substance lighter than a fluid, be immersed in it so that its base does not touch the surface, it will rest in such a position that the axis is perpendicular to the surface… and likewise, if it is immersed so that its base is completely below the surface.*

BOOK 2: concerns the positions of stable hydrostatic equilibrium of a right segment of a paraboloid of revolution of specific gravity less than that of the fluid in which it is immersed, with height H and parameter p for the generating parabola.

Propositions 2/3: *assume $H<(3/4)p$; if the solid is placed in the fluid with its axis at any angle with the vertical, but so that the base is either entirely outside the fluid or entirely submerged, the axis will return to the vertical position.*

Propositions 4/5 give sufficient conditions for the axis to return to the vertical, for a `tall' paraboloid segment ($H> (3/4)p$): *(i) base completely outside the surface of the fluid, ratio of specific gravity of solid to that of fluid greater than the square of $1-(3/4)p/H$; (ii) base completely submersed, ratio of specific gravities smaller than 1 minus the square of $1-(3/4)p/H$.*

Proposition 8: *Assume: (i) $(3/4)p<H<(15/8)p$; (ii) the ratio of specific gravities (solid to fluid) is less than the square of $1-(3/4)p/H$; (iii) the base is entirely outside the surface of the fluid. Then, if the solid is placed in the fluid so that its axis is inclined to the vertical, it will NOT return to the vertical position, and will remain only in the position where its axis makes a certain specific angle with the surface of the fluid.*

Proposition 9: *Assume: (i) $(3/4)p < H < (15/8)p$; (ii) the ratio of specific gravities is greater than 1 minus the square of $(1-(3/4)p/H)$; (iii) the solid is placed in the fluid with its base entirely below the surface. Then the solid, if placed in the fluid with its axis inclined to the vertical, will NOT return to the vertical position, and will remain only in the position where its axis makes a specific angle with the surface of the fluid (the same angle as in Prop. 8.)*

Archimedes goes on to consider practically all the remaining cases of this problem.

Source: T.L Heath, The Works of Archimedes (Dover)

ARCHIMEDES' `*The Sand-Reckoner*'

`THERE are some, king Gelon, who think that the number of the sand is infinite in multitude; and I mean by the sand not only that which exists around Syracuse and the rest of Sicily, but also that which is found in every region, whether inhabited or uninhabited…But I will try to show you by means of geometrical proofs, which you will be able to follow, that of the number named by me and given in the work which I sent to Zeuxippus, some exceed not only the number of the mass of sand equal to the earth filled up [including the highest mountains and all seas and hollows], but also that of a mass equal in magnitude to the entire `universe''.

`…Now, Aristarchus of Samos [c. 270 BC] brought out a book consisting of certain hypothesis, in which the premises lead to the result that the universe is many times greater than that now so called. His hypotheses are that the fixed stars and the sun remain unmoved, that the earth revolves about the sun in the circumference of a circle, the sun lying in the middle of the orbit, and that the sphere of the fixed stars, situated about the same center, is so great…that the ratio the sphere containing the circle on which the earth revolves bears to the sphere containing the fixed stars is the same as the ratio the earth (supposed at the center) bears to what we normally describe as the `universe''.

`I say then, that even if a sphere were made up of sand, as great as Aristarchus supposes the sphere of the fixed stars to be, I shall still prove that, of the numbers named in [my work], some exceed the number of the sand in this sphere, provided the following assumptions are made:'

1. *The perimeter of the earth is about 3,000,000 stadia and not greater*

`Here I am entirely aware that some have tried to prove the perimeter is about 300,000 stadia; so I am even allowing it to be 10 times greater.'

[*Note:* Eratosthenes of Cyrene (a contemporary of Archimedes), by measuring the inclination of the shadow cast by the sun at the summer solstice in Alexandria (5,000 stadia north of Syene and on the same meridian), estimated the circumference at 250,000 stadia (1 stadium~ 0.1 miles), which gives an estimate for the diameter of about 7,850 miles, within 50 miles of the true polar diameter.]

2. *The diameter of the earth is greater than the diameter of the moon, and the diameter of the sun greater than that of the earth.*

3. *The diameter of the sun is about 30 times the diameter of the moon and not greater.*

4. *The diameter of the sun is greater than the side of the chiliagon [regular polygon with 1,000 sides] inscribed in the greatest circle in the sphere of the universe.*

With these assumptions I estimate the diameter of the `universe' to be no more than 10,000 times the diameter of the earth, and therefore at most 10,000,000 stadia.

5. *A quantity of sand not greater than a poppy-seed contains no more than 10,000 grains, while the diameter of a poppy-seed is no less than $1/40^{th}$. Of a finger-breadth.*

[With these assumptions, Archimedes arrives at the estimate of `10,000,000 units of the eighth order of numbers' for the number of grains of sand in the universe, which in modern notation is 10 to the 63^{rd}. power.]

(Source: T.L Heath, The Works of Archimedes (Dover), pp. 221-232.)