MATH 241 — Review problems: Green's theorem, chain rule, change of variables. 4/17/2006

[1] Use Green's theorem to find the counterclockwise circulation and the outward flux for the given $\vec{f}$ and C:

(i) $\vec{f}(x,y) = (x^2+4y)\,\vec{i} + (x+y^2)\,\vec{j}$

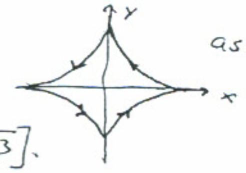
C: square $[0,1]\times[0,1]$.

(ii) $\vec{f}(x,y) = (x+y)\,\vec{i} - (x^2+y^2)\,\vec{j}$

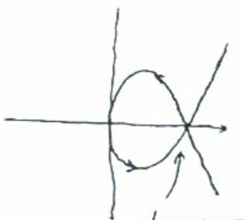
C: triangle bounded by $y=0,\ x=1,\ y=x$

[2] Use Green's theorem to find the area enclosed by the parametrized curves:

(i) $\vec{r}(t) = (\cos^3 t,\ \sin^3 t),\ t\in[0,2\pi]$



astroid



$t = \pm\sqrt{3}$

(ii) $\vec{r}(t) = \left(t^2,\ \frac{t^3}{3} - t\right),\ t\in[-\sqrt{3},\sqrt{3}]$.

[3] Show that the value of $\oint_C xy^2\,dx + (x^2y+2x)\,dy$ around the boundary of a square depends only on the area of the square, not on its position.

[4] The line integral of the vector field $\vec{f}(x,y) = (x, xy)$ is zero for every circle C with center at the origin. However, this vector field is not a gradient. Verify. How is this possible? (Why doesn't it contradict Green's theorem?)

[5] (i) Find $\frac{\partial z}{\partial u}$ when $u=0, v=1$ if $z = \sin(xy) + x\sin y$, $x = u^2+v^2$, $y = uv$

(ii) Find z_x and z_y at $(2,3,6)$ if $\frac{1}{x}+\frac{1}{y}+\frac{1}{z} = 1$ for all (x,y,z).

[6] Show that if $f(u,v)$ satisfies the Laplace equation $f_{uu}+f_{vv}=0$ and $u = \frac{1}{2}(x^2-y^2)$, $v = xy$, then w satisfies the Laplace equation $w_{xx}+w_{yy}=0$. Here $w(x,y) = f(u(x,y), v(x,y))$.

[7] Use a change of variables to show: $\iint_Q f(x+y)\,dx\,dy = \int_{-1}^{1} f(u)\,du$. $Q=\{(x,y)\,|\,|x|+|y|\le 1\}$

[8] Evaluate $\int_0^{2/3}\int_y^{2-2y} (x+2y)\,e^{(y-x)}\,dx\,dy$ using $\begin{cases} u = x+2y \\ v = x-y\end{cases}$ to express it as an double integral in (u,v). (You will need the inverse mapping $(u,v)\mapsto(x,y)$).