

LIST 5-double integrals, surface area, volume

1. Find the integral of f over the given region D . Sketch the region and indicate the 'slicing' used.

$$\int \int_D x^3 y^2 dA, \quad D = \{0 \leq x \leq 2, -x \leq y \leq x\}.$$
$$\int \int_D x + y dA, \quad D \text{ bounded by } y = \sqrt{x}, y = x^2.$$

2. Evaluate the double integral- sketch the region and 'slicing' used. (see above)

3. Sketch the region and evaluate the double integral, reversing the order of integration if necessary.

$$\int_0^1 \int_{\sqrt{x}}^1 e^{y^3} dy dx$$
$$\int_0^8 \int_{x^{1/3}}^2 \frac{1}{y^4 + 1} dx dy$$

4. Find the volume of the solid region described below:
under surface $z = xy$, above triangle $(1, 1), (4, 1), (1, 2)$.
enclosed by paraboloid $z = x^2 + 3y^2$ and planes $x = 0, y = 1, y = x, z = 0$.

5. Change into an integral in polar coordinates and evaluate:

$$\int_{-1}^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} x dx dy$$
$$\int_0^1 \int_0^{\sqrt{9-x^2}} \frac{2}{(1+x^2+y^2)^2}$$

6. Use polar coordinates to find the volume of the solid described below.
Below paraboloid $z = 18 - 2x^2 - 2y^2$, above xy plane;
Bounded by paraboloid $z = 1 + 2(x^2 + y^2)$ and plane $z = 7$; in first quadrant.

7. Sketch the region $D \subset \mathbb{R}^2$ and compute the coordinates of its centroid.

region bounded by $x = 0, y = z, y = 2 - x^2$ (1st. quadrant)
region between the x-axis and the arch $y = \sin x, 0 \leq x \leq \pi$.

8. Write parametric equations for the surface given below, and use the parametrization to compute its area.

band cut from the paraboloid $z = x^2 + y^2$ by the planes $z = 2, z = 6$.
cap cut from the sphere $x^2 + y^2 + z^2 = 4$ by the cone $z = \sqrt{x^2 + y^2}$.

9. Use a parametrization to compute the integral of the given function $f(x, y, z)$, over the given surface.

$f(x, y, z) = z$, over $y^2 + z^2 = 4, z \geq 0, 1 \leq x \leq 4$.

$f(x, y, z) = z^2$ over hemisphere $x^2 + y^2 + z^2 = R^2, z \geq 0$.

10. Find the area of the surface described below, *without* using a parametrization.

(same surfaces as in problem 8)

11. Compute the surface integral of the given function over the given surface, without using a parametrization.

$f(x, y, z) = y + z$; surface of wedge in first octant bounded by the coordinate planes and the planes $x = 2, y + z = 1$.

centroid of the surface cut from the cylinder $y^2 + z^2 = 9, z \geq 0$ by the planes $x = 0, x = 3$.