

Math 241, Spring 2006- EXAM 1. NAME:

Instructions. Closed book, closed notes, no calculators. No credit for answers without justification- show all work. 50-min test.

1. (*List 1:3,5*) [10] For the parameterized helix $\mathbf{r}(t) = (12t, 5 \sin t, 5 \cos t)$, $t \in [0, 2\pi]$, find: (1) the total length; (ii) parametric equations for the tangent line at the point $(4\pi, 5\sqrt{3}/2, 5/2)$

2. (*List 1:10*) [10] A particle moves with *constant* speed $v = 3$ on the ellipse $x^2 + 2y^2 = 12$ (counterclockwise). When the particle is at the point $(2,-2)$, find: (i) the velocity vector; (ii) the direction of the acceleration vector (recall directions are given by *unit* vectors).

3. (*List 3:8*) [5] Let $f(x, y) = x^3 + y^3 + 6xy$. The origin is a critical point. Let $\mathbf{r}(t)$ be a smooth curve satisfying $\mathbf{r}(0) = (0, 0)$, $\mathbf{r}'(0) = (1, 1)$. Does the function of one variable $g(t) = f(\mathbf{r}(t))$ have a local max, local min or an inflection point when $t = 0$?

4. (*List 3:5*) [5] Find a unit normal vector to the hyperboloid $2x^2 + 2y^2 - 3z^2 = 1$, at the point $(1, 1, 1)$.

5. (*List 2:8*) [5] Let $f(x, y, z) = x^{-1}y^5z^3$ ($x \neq 0$); assume the values of x, y, z are known up to an error, so that $|\frac{\Delta x}{x}| \leq 0.005$, with the same relative error estimates for y and z . Estimate the relative error $|\frac{\Delta f}{f}|$ in the computed value of f .

6. (*List 3:7, 14*) [10] Let $f(x, y) = x^4 + y^4 + 4xy$. (i) Locate a saddle point of f (if there is one); (ii) Write down the degree 2 Taylor polynomial of f at $(-1, -1)$.

7. (*Lecture: 2/10*) [5] Explain why the function in problem 6 is guaranteed to have an absolute (global) minimum, over all $(x, y) \in \mathbb{R}^2$.