Math 241, Spring 2006 (Alex Freire)

LIST 3: Tangent planes, Taylor approximations, classification of critical points, max/min.

1. The temperature at the point (x, y) is $T(x, y) = \cos(3 \cdot x + 5 \cdot y)$. If you are at the point $A = (1, 4)$ and you can go only with speed 4 in the direction of either $P(6, 5)$ or $Q(5, 7)$, in which direction is the increase of the temperature the highest ? (Answer : toward Q)

2. For the given function of two variables f and point P, find (i) the first-order Taylor approximation at P; (ii) an equation for the tangent plane to the graph of f at P.

$$f(x, y) = x^3y^4, \quad P(1, 1)$$

3. Find the linearization (first-order Taylor approximation) of the function of 3 variables f, at the given point P.

$$f(x, y, z) = xy + 2xz + 3yz, \quad P(1, 1, 1)$$

4.Find the equation $y = m \cdot x + b$ of the tangent line to the ellipse $2 \cdot x^2 + y^2 = 9$ at $(2, 1)$. (Answer : $y = -4 \cdot x + 9$)

5. Find a unit normal $\vec{n}$ to the surface $3 \cdot x^2 - 4 \cdot y^2 + 5 \cdot z^2 = -92$, at the point $P(1, 5, 1)$. (Answer : $[0.14, -.96, 0.24]$)

6. Find the maximum of the function $f(x, y) = 2 \cdot y + 1 \cdot x^2 + 3 \cdot xy - 1 \cdot y^2 - 16 \cdot x$ on the segment joining $P(4, 2)$ and $Q(1, 5)$. (Answer : -12)

7. Find a a quadratic approximation for the given function of two variables near the origin (second-order Taylor formula)

$$f(x, y) = xe^y$$

8. $(4, 4)$ is a critical point of $f(x, y) = 2 \cdot x^2 + 3 \cdot x \cdot y + 1 \cdot y^2 - 28 \cdot x - 20 \cdot y$. Is $t = 0$ a local minimum or a local maximum of $f(4 + 3 \cdot t, 4 - 4 \cdot t)$? (Answer : local maximum)

9. Let $f(x, y) = 4 \cdot x^2 + 6 \cdot x \cdot y + 2 \cdot y^2 - 28 \cdot x - 20 \cdot y$. Classify all critical points of f. (Answer : saddle point at $(x, y) = (2, 2)$)

10. Let $f(x, y) = 37 \cdot x + 41 \cdot y - 3 \cdot x^2 - 7 \cdot x \cdot y - 3 \cdot y^2$. Classify all critical points of f. (Answer : saddle point at $(x, y) = (5, 1)$)

11. On the lines, one passing through points $P(0,0,0)$ and $Q(0,0,1)$ and the other passing through points $R(-1,2,4)$ and $S(2,3,3)$, find points that are closest to each other. (Answer : $(0,0,3.9),(-.7,2.1,3.9)$)

12. Find the absolute minimum of $f(x,y) = (3\cdot x+y+3)^2+(5\cdot x+y+3)^2+(-5\cdot x+y-3)^2$. (Answer : 0.86)

13. Let $f(x,y) = x^3-675\cdot x+x^2\cdot y^2+90\cdot x\cdot y$. Find all local minima of f. (Answer : $(15,-3)$ is a local minimum)

14. Let $f(x,y) = x^3-300\cdot x+x^2\cdot y^2+40\cdot x\cdot y$. Find all saddle points of f. (Answer : $(0,7.5)$ and $(-10,2)$ are saddle points)

15.(Review) For the given parametrized space curve and given value of t, compute (i) the decomposition of the acceleration vector into tangential and normal components; (ii) the speed; (iii) the radius of curvature.

$$\mathbf{r}(t) = (t, t^2, t^3), \quad t = 1$$