Homework problems- variation of constants and mechanics.

Use 'variation of constants' to find the general solution of the following non-homogeneous equations; two solutions of the associated homogeneous equation are given:

1. $t^2y'' - ty' + y = t; \quad y_1(t) = t, \quad y_2(t) = t\ln t. \quad Ans. y_p(t) = \frac{t}{2}(\ln t)^2$

2. $2t^2y'' + 3ty' - y = t^{-1}; \quad y_1(t) = t^{1/2}, \quad y_2(t) = t^{-1} \quad Ans. y_p(t) = -\frac{1}{3}t^{-1}\ln t$

3. A particle moves on a straight line according to the equation of motion:

$$y'' + 4y' + 13y = 0, \quad y = y(t).$$

At $t = 0$, $y = 0$ and $y' = 12$. (1) Find the solution $y = y(t)$. (2) Find the time required for the damped amplitude to decrease by 50%.

4. A particle moves on a straight line in accordance with the equation:

$$y'' + 10y' + 16y = 0, \quad y = y(t).$$

At $t = 0$, $y = 1$ and $y' = 0$. (1) Find the solution $y(t)$. (2) When does y attain its maximum value? Does the particle reach $y = 0$ for $t > 0$? (3) Draw a graph of the motion (y vs. t). Is the motion oscillatory?

5. A particle of mass m moves on a straight line. It is attracted toward the origin by a force equal to k times its distance from the origin. The resistance is $2R$ times the velocity. Find the maximum value of m so that the motion will *not* be oscillatory.

6. A 10 kg body stretches a spring 5 cm. After it is brought to rest, it is displaced a further 6 cm, attached to a driving force $f(t) = \sin(4\sqrt{6}t)$ N and given a downward velocity of $4m/s$. Find the equation of motion $y(t)$, where y is displacement from equilibrium. Is the motion bounded or unbounded? What is the undamped resonant frequency?

7. A particle weighing 16 lb moving on a horizontal line is attracted to the origin 0 by a force proportional to the distance to 0. When the particle is at $y = -2$, this force is 9 lb. In addition, a forcing function $f(t) = \sin 3tA$ is impressed on the system. If at $t = 0$, $y = 2$ and $y' = 0$, find (1) the equation of motion $y(t)$ and (2) the resonant frequency of the system.

8. A particle moves in accordance with the law:

$$y'' + 4y' + 16y = f(t), \quad y = y(t).$$

(1) What frequency ω_f of the function $f(t)$ will make the period of the steady-state motion equal to $\pi/3$? (2) What frequency ω_f of $f(t)$ will produce resonance (i.e., maximum amplification factor)? (3) What is the value of the amplification factor at resonance?

9. A 16 lb weight stretches a spring 6 in. Its coefficient of damping is 2. The 16lb weight is removed, replaced by a 64 lb weight and brought to rest. At $t = 0$, a forcing function $8\sin(t)$ is applied to the system. Find the steady-state motion and the amplification factor of the system.