SEPARABLE EQUATIONS AND THE EXISTENCE-UNIQUENESS THEOREM

A. Examples discussed in class.

1. $y' = \frac{t^2}{y^2}$ (all solutions defined on a half-line)
2. $e^y y' - (t+t^3) = 0$ (solutions to the IVP $y(0) = y_0$ defined for all time, but some solutions defined only on a half-line)
3. $y' = 1 + y^2$ (blowup in finite time)
4. $yy' + (1+y^2)\sin t = 0$ (finite existence time without blowup)
5. $y' = (1-y)t, \quad y(0) = 1$ (solution of IVP is a constant solution)
6. $y' = -\frac{t}{y}$ (solution curves contained in circles)
7. $2tyy' = 3y^2 - t^2$ (homogeneous equation)

B. Proposed problems (homework)

In 1-4, find the general solution ($y = y(t)$)

1. $y' = \frac{1-t^2}{y^2} \qquad Ans. \quad y(t) = (3t - t^3 + C)^{1/3}$
2. $y' = y(2 + \sin t) \qquad Ans. \quad y(t) = Ce^{2t - \cos t}$
3. $y' + y^2 = y \qquad Ans. \quad y(t) = Ce^t/(Ce^t - 1)$ and $y(t) \equiv 1$
4. $y' = \frac{t+y}{t-y}$(homogeneous) Ans. $e^{\arctan(y/t)} = Ct(1 + y^2/t^2)^{1/2}$ (implicit)

In 5-8, solve the initial-value problem for $y = y(t)$. Include the interval where the solution is defined and a plot of the solution.

5. $y' = \frac{2t}{y + yt^2}, \quad y(2) = 3 \qquad Ans. \quad y(t) = (9 + 2\ln((\frac{1+t^2}{5})]^{1/2}$
6. $(1-t)y' = t(y+1), \quad y(0) = 0 \quad Ans. \quad y(t) = -1 + e^{-t}/(1-t).$
7. $y' = \frac{3t^2+4t+2}{2(y-1)}, \quad y(0) = -1 \qquad Ans. \quad y(t) = 1 - (4 + 2t + 2t^2 + t^3)^{1/2}$
8. $(t - \sqrt{ty})y' = y, y(1) = 1$(homogeneous) Ans $y(t) = e^{-2(\sqrt{t/y} - 1)}$(implicit)

(The answers for 5-8 are not complete-the domain is missing.)