MATH 231- SECOND EXAM- NOVEMBER 6, 2003

Instructions. Solve the problems below. Closed book, closed notes; all formulas needed are given on the exam. Calculators not allowed. Time given: 70 minutes (11:10 to 12:20). **L** denotes Laplace transform.

1.[10] Use Laplace transforms and Heaviside's formula $\mathbf{L}^{-1}(P/Q) = \sum \frac{P(r_i)}{Q'(r_i)} e^{r_i t}$ to solve the IVP:

$$y'' - 3y' + 2y = e^{3t}, \quad y(1) = y'(1) = 0.$$

(Note the ICs are given at $t = 1$.)

2.[20] (i) Solve the IVP:

$$y'' + 4y = f(t), \quad y(0) = y'(0) = 0, \quad f(t) = 1 - u(t - \pi),$$

where $u(t)$ is the "unit step function at 0".

(ii) Sketch the graph of the solution on the interval $[0, 2\pi]$.

Given: $\mathbf{L}[u(t-a)] = \frac{e^{-as}}{s}$,$\mathbf{L}^{-1}[e^{-as}F(s)] = f(t-a)u(t-a)$.

3.[40] Consider the system of first-order DEs:

$$x_1' = x_1 + x_2, \quad x_2' = 4x_1 + x_2.$$

The system may be written in vector form as $x' = Ax$, where A is a 2 X 2 matrix with eigenvalues -1 (with $(1, -2)$ as an eigenvector) and 3 (with (1,2) as an eigenvector).

(i) Write down the general solution (in vector form, as column vectors);

(ii) sketch the trajectory in the phase plane of the solution with ICs $x_1 = 1, x_2 = 0$, for $t \geq 0$;

(iii) sketch the graph of x_1 vs. t for $t \geq 0$, for the same solution;

(iv) write down the fundamental matrix $\Phi(t)$ with $\Phi(0) = I$.

4. [10] Use the "variation of parameters formula" $x_p(t) = \Phi(t)v(t), v'(t) = \Phi(-t)f(t)$ to find a particular solution for the system:

$$x_1' = x_1 + x_2 + 2, \quad x_2' = 4x_1 + x_2 - 4.$$

(*Remark:* you may also solve this by reducing to a second order equation for x_1- your choice.)

5. [10] Show that $E(x_1, x_2) = x_1^2 - 2x_1x_2 + 2x_2^2$ is a conserved quantity for the system:

$$x_1' = 2x_1 - 4x_2, \quad x_2' = 2x_1 - 2x_2.$$