Differential Equations-1st. Exam- October 2, 2003

Instructions. Solve the following problems. No credit for answers without justification. Closed book, closed notes. Calculators not allowed. Good luck!

Time given: 75 min (11:10-12:25.)

1. [12 pts. ea.] Find the general solution \( y = y(t) \) in each case:
   \( a. \) \( y' + t^2 y = t^2; \)
   \( b. \) \( 2tyy' + y^2 + 2t = 0 \) (exact);
   \( c. \) \( y' - 2y - 5y^3 = 0 \) (Bernouilli);
   \( d. \) \( y'' + 4y = \sin t; \)
   \( e. \) \( t^2y'' - 3ty' + 3y = t, \quad t > 0 \) (homogeneous eqn. is of Euler type).

2. [7 pts. ea.] Find the largest interval where the solution of the initial-value problem is defined; justify your answer. \( y = y(x) \) in each case.
   \( a. \) \( (\cos x)y' + (\ln x)y = 2x, \quad y(1) = 1; \)
   \( b. \) \( (x - 2)y'' + xy' + y = 0, \quad y(1) = y'(1) = 0; \)
   \( c. \) \( y' + (y - 1)(y - 2) = 0, \quad y(0) = 3/2. \)

3. For the autonomous first-order equation:

\[ y' = y - y^{1/3}, \quad y = y(t): \]

(items can be answered independently)

\( a. \) [3 pts.] Find the constant solutions;
\( b. \) [10 pts.] Find the general solution, assuming \( y(0) > 1. \) (Hint: let \( v = y^{1/3} \));
\( c. \) [6 pts.] The functions \( y_1(t) \equiv 0 \) (for all \( t \)) and:

\[ y_2(t) = 0 \text{ for } t \leq 0, \quad y_2(t) = (1 - e^{2t/3})^{3/2} \text{ for } t \geq 0 \]

both solve the initial-value problem defined by the equation and the initial condition \( y(0) = 0. \) Why doesn't this contradict the existence-uniqueness theorem for first-order equations?