Math 231, Section 4, Problem Set 1

Due Wednesday, Jan. 20, 2010

From Text *Fundamentals of Differential Equations*, by Nagle, Saff, and Snider

Section 1.1, # 13, 14, 15, 16
Section 1.2, # 3, 5, 14, 20

Additional Problems:

1.) Look up “Navier-Stokes equations” on Wikipedia and read enough to answer the following questions.
   A.) Write down the general form of the Navier-Stokes equations.
   B.) What does the unknown (usually denoted \( v \)) in the Navier-Stokes equations represent physically?
   C.) If you are the first to prove or disprove the global existence and uniqueness of solutions to the Navier-Stokes equations, you will win a cash prize from the Clay Mathematics Institute. How much is the prize?

2.) Check whether \( y = 2t^2 + t^3 \) is a solution of \( ty' - y = t^2 \).

3.) Check whether \( y = Ce^{-x} + x - 1 \) is a solution of \( y' = x - y \).

4.) Show that \( y = \frac{1}{1-x^2} \) is a solution of \( y' - 2xy^2 = 0 \), \( y(0) = 1 \). What happens to this solution as \( x \to 1^- \) (that is, \( x \) goes toward 1 from the left) or \( x \to -1^+ \) (that is, \( x \) goes to \(-1 \) from the right)?

5.) Solve \( y' = x + e^{4x} \).

6.) A.) Solve \( y' = \frac{x + 3}{x^2 + 3x + 2} \).

   Hint: To integrate, use the method of Partial Fractions. To integrate \( \frac{p(x)}{q(x)} \), where \( p \) and \( q \) are polynomials and the degree of \( p \) is strictly less than the degree of \( q \), first try to factor \( q \). If \( q \) factors completely into terms of degree 1 (that is, into \( (x - a)(x - b) \cdots \)), then one can find numbers \( A, B, \ldots \) so that
   \[
   \frac{p(x)}{q(x)} = \frac{A}{x - a} + \frac{B}{x - b} + \cdots.
   \]

   To find the numbers \( A, B, \ldots \), multiply through the equation on both sides by the product of all of the terms \( (x - a), (x - b), \ldots \). Then set \( x = a \) to find \( A \), \( x = b \) to find \( B \), etc. Once you know these numbers, you have the “partial fraction expansion” of \( p/q \) and you can use that identity to do the integration. I will remind you in class what to do if \( q \) does not factor completely into a product of monomial terms. We will use this method a lot in Chapter 7.

   B. Solve \( y' = \frac{x + 3}{x^2 + 3x + 2} \), \( y(0) = 0 \).  

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7.) Solve $y' = \frac{x}{x + 1}$.

Hint: To integrate, either write $x = x + 1 - 1$ in the numerator and then separate into two terms, or divide the denominator into the numerator to get a non-fractional term (in this example an integer) plus a fraction coming from the remainder, where now in the fraction the numerator has lower degree than the denominator. The second method applies more generally to $p(x)/q(x)$ where $p$ and $q$ are polynomials and the degree of $p$ is equal to or greater than the degree of $q$. Then the method of partial fractions is used on the term coming from the remainder.

8.) A.) Solve $y' = xe^{2x}$.

Hint: remember the integration by parts formula

$$\int u dv = uv - \int v du.$$ 

B.) Solve $y' = xe^{2x}$, $y(1) = 2$. 