NAME: ____________________________

#1.) _______/20  #2.) _______/20  #3.) _______/10  #4.) _______/10  
#5.) _______/20  #6.) _______/20  Total: _______/100

Instructions: There are 5 pages and a total of 100 points on the exam. You must show all necessary work to get credit. You may not use your book, notes, or calculator. Partial credit will only be given for progress toward a correct solution.

1.) (20 points) Solve \( y'' - 6y' + 13y = 60 \cos t \).
2.) a.) (5 points) Check that the functions $y_1 = t^2$ and $y_2 = t^{-1}$ are solutions of
\[ (*) \quad y'' - \frac{2}{t^2}y = 0, \quad \text{for } t > 0. \]

b.) (15 points) Find the general solution of
\[ y'' - \frac{2}{t^2}y = 3t^3, \quad \text{for } t > 0. \]

Warning: The method of undetermined coefficients does not apply to variable coefficient equations.
3.) (10 points) Solve the Cauchy-Euler equation \( t^2 y'' + ty' + 4y = 0 \), for \( t > 0 \).

4.) (10 points) Solve \( y'' - 2y' = 4e^{2t} \).
5.) (20 points) A spring has mass 2 kg, damping constant 12 N·sec/m, and spring constant (stiffness) 16 N/m. If it is stretched 1 m beyond its natural length and given an initial velocity of 2 m/sec in the same direction it is stretched, find the time in seconds when the spring reaches its maximum displacement from equilibrium. There is no external force acting on the spring (after the initial velocity is given).
6.) (20 points) Assuming the fact that the function $y_1 = e^t$ is a solution of the equation

\[
(*) \quad y'' - \left(2 + \frac{1}{t}\right) y' + \left(1 + \frac{1}{t}\right) y = 0,
\]

find the general solution of ($*$).