5. **1)** [25 points] Let $P \mid Q$ denote "P and Q are not both true".

(a) Write the truth table of $P \mid Q$.

Solution. The truth table is:

P	Q	$P \mid Q$
T	T	F
T	F	T
F	T	T
F	F	F

$\square$

(b) Find a formula [involving P and Q] using only $\wedge$, $\vee$ and $\neg$ operations logically equivalent to $P \mid Q$.

Solution. $\neg(P \wedge Q)$ or $(\neg P) \vee (\neg Q)$. $\square$

(c) Find a formula logically equivalent to $\neg P$ using only $\mid$ [and P]. [*Show* that your formula is indeed equivalent!]

Solution. We have $P \mid P$ works:

$$P \mid P \sim \neg(P \wedge P) \sim \neg P.$$

$\square$

(d) Find a formula for $P \wedge Q$ using only $\mid$ [and P and Q]. [*Show* that your formula is indeed equivalent!]

Solution. We have that $P \mid Q$ is $\neg(P \wedge Q)$ by (b). So by double negatives, we need to negate $P \mid Q$ to get $P \wedge Q$. But, by (c), this is the same as $(P \mid Q) \mid (P \mid Q)$. $\square$

(e) Find a formula for $P \vee Q$ using only $\mid$ [and P and Q]. [*Show* that your formula is indeed equivalent!]

Solution. By DeMorgan's Law, $P \mid Q \sim (\neq P) \vee (\neg Q)$. So, $(\neg P) \mid (\neg Q) \sim P \vee Q$ [by double negatives]. By part (c), we then have $(P \mid P) \mid (Q \mid Q) \sim P \vee Q$. $\square$