February 19th, 2007

Math 456

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Spring 2007

Name: ....

Student ID (last 5 digits): XXXX................

Midterm 1

You have 50 minutes to complete the exam. Do all work on this exam, i.e., on the page of the respective assignment. Indicate clearly, when you continue your solution on the back of the page or another part of the exam.

Write your name and the last five digits of your student ID number on the top of this page. Check that no pages of your exam are missing. This exam has 4 questions and 6 printed pages (including this one and a page for scratch work in the end).

No books, notes or calculators are allowed on this exam!

Show all work! Even correct answers without work may result in point deductions. Also, **points will be taken from messy solutions**.

Good luck!

Question	Max. Points	Score
1	25	
2	25	
3	25	
4	25	
Total	100	

1) Let R be a ring and I be an ideal of R.

(a) Prove that if J is an ideal of R containing I, then $\bar{J} \stackrel{\text{def}}{=} \{\bar{a} \in R/I \ :\ a \in J\}$ is an ideal of R/I.

(b) Prove that if $\bar{J}'$ is an ideal of R/I, then $J' \stackrel{\text{def}}{=} \{a \in R \ :\ \bar{a} \in \bar{J}'\}$ is an ideal of R containing I.

2) Let R be a commutative ring with identity and $a \in R$ such that $a^{n-1} \neq 0$, but $a^n = 0$, for some positive integer n. Prove that $R[x]/(ax-1) = \{\bar{0}\}$, i.e., it is the *zero ring*.

3) Let R be an integral domain, F be its field of fractions [or quotient field], and K be field such that $R \subseteq K$. Prove that there is an *injective homomorphism* $\phi : F \to K$, such that for all $a \in R$, $\phi\left(\frac{a}{1}\right) = a$. [**Hint:** To start, you need to find the formula for ϕ. Think of the most natural way of seeing an element of F inside of K, remembering that the image is contained in a *field*. Also, you will have to show that your formula is well defined, i.e., if $\frac{a}{b} = \frac{c}{d}$, then $\phi\left(\frac{a}{b}\right) = \phi\left(\frac{c}{d}\right)$.]

4) Prove that $\mathbb{Z}[\mathrm{i}\sqrt{3}]/(2-\mathrm{i}\sqrt{3}) \cong \mathbb{Z}/7\mathbb{Z}$.

Scratch: