Problem from this year’s “UT Math Contest” (Fermat II) for high school students.

You are not allowed to talk to anyone about this problem! (At least until it is due.)

**Problem:** Let \( a, b, c \in \{1, 2, \ldots, 2005\} \) and

\[
f(X) \overset{\text{def}}{=} aX^{101} + bX^{100} + c.
\]

Prove that if \( f(2006) \) is prime, then \( f(X) \) has no integral root, i.e., there is no \( n \in \mathbb{Z} \) such that

\[
a n^{101} + b n^{100} + c = 0.
\]