Exam #2 Math 32 – Fall 2005

Name:

I have adhered to the Duke Community standard in completing this test.

Signature

Q 1 /10
Q 2 /18
Q 3 /15
Q 4 /30
Q 5 /15
Q 6 /12
Total: /100

Here’s a list of formulas that may be of use to you. Good luck!

• \( \frac{d}{dx} \sinh^{-1}(x) = \frac{1}{\sqrt{1+x^2}}, \frac{d}{dx} \cosh^{-1}(x) = \frac{1}{\sqrt{x^2-1}}, \frac{d}{dx} \tanh^{-1}(x) = \frac{1}{1-x^2} \)

• If an integral involves \( a^2 - u^2 \), substitute \( u = a \sin(\theta) \).
  If an integral involves \( a^2 + u^2 \), substitute \( u = a \tan(\theta) \).
  If an integral involves \( u^2 - a^2 \), substitute \( u = a \sec(\theta) \).
1. Solve the following initial value problems:
   (a) (5 pts.) \( \frac{dy}{dx} = x^4 + y^2 x^4, \quad y(0) = 1 \).

   (b) (5 pts.) \( \frac{dy}{dx} = \frac{y}{x \ln(x)}, \quad y(e) = 2 \).

2. Compute the following limits.
   (a) (6 pts.) \( \lim_{x \to 0^+} x^x = \)
(b) (6 pts.) \( \lim_{x \to \infty} \frac{\tan^{-1}(x)}{e^x} = \) 

(c) (6 pts.) \( \lim_{x \to \infty} \left( \frac{e^x - e^{-x}}{e^x + e^{-x}} \right)^x = \)

3. Compute the following integrals:

(a) (5 pts.) \( \int \frac{\sinh^3(x)}{\cosh^3(x)} \, dx = \)
4. (a) (10 pts.) Write a factorization of \((x^3 + 1)\) as a product of a linear term and a quadratic term (Hint: find a root, then use long division).
(b) (10 pts.) Write a partial fraction decomposition of \( \frac{1}{x^3+1} \)

(c) (10 pts.) Compute the integral \( \int \frac{1}{x^3+1} \, dx \)
5. (a) (5 pts.) Prove that \( \csc(x) = \frac{\sin(x)}{1-\cos^2(x)} \)

(b) (5 pts.) Use part (a) and the fact that \( \frac{1}{1-u^2} = \frac{1}{2} \left( \frac{1}{1+u} + \frac{1}{1-u} \right) \) to prove that \( \csc(x) = \frac{1}{2} \left( \frac{\sin(x)}{1+\cos x} \right) + \frac{1}{2} \left( \frac{\sin(x)}{1-\cos(x)} \right) \)

(c) (5 pts.) Compute \( \int \csc(x) \, dx \)
6. Determine whether or not the following improper integrals converge. Evaluate those that do converge.

(a) (6 pts.) \( \int_{\pi}^{\infty} \frac{1}{(x-3)^{\frac{3}{2}}} \, dx \)

(b) (6 pts.) \( \int_{0}^{\pi/2} \tan(x) \, dx \)