

20 points

Group No: \_\_\_\_\_

Limit notation: Know when to use  $\lim(\ )$  and when to drop it.

1. Which of the following represent correct use of mathematical notation? Select all that apply.

X A.  $\lim_{x \rightarrow 3} \left( \frac{x^2 - 9}{x - 3} \right) = x + 3 = 6$

X B.  $\lim_{x \rightarrow 3} \left( \frac{1 - x^2}{5} \right) = \frac{1 - 3^2}{5} = \frac{-8}{5} = \frac{-8}{5}$

X C. Given:  $f(x) = \begin{cases} x - 5 & x \leq 5 \\ x^2 - 1 & x > 5 \end{cases}$

In finding left and right hand limits a student only shows:  $5 - 5 = 0$ ;  $5^2 - 1 = 24$  and states so the limit DNE.

Need  $\lim_{x \rightarrow 5^-} (x - 5) = 5 - 5 = 0$   
 $\lim_{x \rightarrow 5^+} (x^2 - 1) = 5^2 - 1 = 24$

X D.  $\lim_{x \rightarrow 5} = 14$

limit (sand)  
 e. None!

2. Which of the following represent correct use of mathematical notation? Select all that apply.

A. Given:  $f(x) = \begin{cases} x - 5 & x \leq 5 \\ x^2 - 1 & x > 5 \end{cases}$

In finding the left and right hand limits a student shows:

$\lim_{x \rightarrow 5^-} (x - 5) = 5 - 5 = 0$  ✓

$\lim_{x \rightarrow 5^+} (x^2 - 1) = 5^2 - 1 = 24$  and states the

$\lim_{x \rightarrow 5} f(x) = DNE$  ✓

B.  $\lim_{x \rightarrow 5} (14) = 14$  ✓

C.  $\lim_{x \rightarrow 1} \frac{2x^2 - x - 1}{x - 1} = \lim_{x \rightarrow 1} \frac{(2x + 1)(x - 1)}{x - 1} = \lim_{x \rightarrow 1} (2x + 1) = 2(1) + 1 = 3$  ✓

D. All of these

4

Limit definition of derivative: Consider this problem: Use the limit definition of derivative to find  $f'(x)$  for  $f(x) = 3x^2 - 4$ .

Each of the following represents incorrect use of mathematical notation. Write a corrected version of each step in the definition of derivative problem shown. (The math is correct, but there is something wrong with the notation used in each line of the process.)

A.  $f'(x) = \lim_{\Delta x \rightarrow 0} \frac{3(x + \Delta x)^2 - 4 - 3x^2 - 4}{\Delta x}$

Corrections (3):  
 \* drop f  
 \* parentheses  $(3x^2 - 4)$   
 \* no =

$f'(x) = \lim_{\Delta x \rightarrow 0} \frac{3(x + \Delta x)^2 - 4 - (3x^2 - 4)}{\Delta x}$

B.  $= \lim_{\Delta x \rightarrow 0} f'(x) \frac{3(x^2 + 2x\Delta x + \Delta x^2) - 4 - (3x^2 - 4)}{\Delta x}$

Correction: \* No  $f'(x)$

C.  $= \lim_{\Delta x \rightarrow 0} \frac{3x^2 + 6x(\Delta x) + 3(\Delta x)^2 - 4 - 3x^2 + 4}{\Delta x}$

Correction: \*  $\lim_{\Delta x \rightarrow 0} \frac{3x^2 + 6x(\Delta x) + 3(\Delta x)^2 - 4 - 3x^2 + 4}{\Delta x}$

D.  $= \lim_{\Delta x \rightarrow 0} \frac{\Delta x(6x + 3\Delta x)}{\Delta x} = 6x + 3\Delta x$

Correction: \* Need  $\lim_{\Delta x \rightarrow 0}$

E.  $= \lim_{\Delta x \rightarrow 0} (6x + 3\Delta x) = \lim_{\Delta x \rightarrow 0} 6x + 3(0)$

Correction: \* Don't need  $\lim_{\Delta x \rightarrow 0}$

F.  $= \lim_{\Delta x \rightarrow 0} = 6x$

Correction: \* No  $\lim_{\Delta x \rightarrow 0} = 6x$

Derivative Notation: A function is not equal to its derivative. Functions and derivatives should be labeled appropriately.

Each of the following represents incorrect use of mathematical notation. Write a corrected version of each.

A.  $f(x) = \frac{x^3}{9} \neq \frac{1}{9}(3x^2)$

$f'(x) = \frac{1}{9}(3x^2)$   
 ①

B.  $f'(x) = \frac{1}{3}x^2 \neq \frac{1}{3}(3)^2$

$f'(3) = \frac{1}{3}(3)^2$   
 ①

C.  $y = x^3(-2x^4 + 6) = -2x^7 + 6x^3 \neq -14x^6 + 18x^2$

$y' = -14x^6 + 18x^2$   
 ①