OPTIMUM HARVEST IN THE RICKER SPAWNER-RECRUIT MODEL:

Background:

An important issue in fisheries management (and in the management of other natural resource, such as trees) is determining how many fish can be caught (harvested) without causing the population to go extinct. Each year a number of adult fish (known as *spawners*) give birth to offspring. These offspring will mature over the next year and then return to the spawning ground to reproduce the next year. Those that return to reproduce are known as *recruits*. As long as the number of recruits is greater than the number of spawners, fishers may catch (harvest) the excess fish, leaving the same number of spawners as the year before. In this way, the size of the fishery population remains constant from year-to-year.

The number of recruits each year depends on the number of spawners from the previous year and on several biological and ecological factors. In mathematical terms, the number of recruits is a function of the number of spawners, and based on several hypotheses, it takes the following form:

$$R(S) = bSe^{-mS}$$

where

R is the number of recruits (in thousands),

S is the number of spawners (in thousands),

b is a measure of the birth rate of the population (how many offspring are born to each spawner),

and m is a measure of the mortality of the offspring.

m and b are both constants that depend on biological and ecological factors.
(See pg. 156-157 and 195-196 in the textbook for more details on where this function and spawner-recruit models in general come from.)

As long as the number of recruits exceeds the number of spawners, the following function describes how many fish may be harvested each year:

$$H(S) = R(S) - S = bSe^{-mS} - S$$

You must use Maple for the following:

1. Find the value of S that maximizes $R(S)$ and find the corresponding maximum value of $R(S)$. (Notice that this value of S should not depend on the birth rate, b.)
2. Make a graph of $R(S)$ when $b = 3$ and $m = 0.1$. $(0 \le S \le 50)$
3. Find the value of S that maximizes $R(S)$ when $b = 3$ and $m = 0.1$.
4. For what range of spawner population size can the population be harvested? (Hint: Find where $H(S) > 0$)
5. What is the range when $b = 3$ and $m = 0.1$?
6. Show that the value of S that maximizes $H(S)$ satisfies the equation: $be^{-mS}(1 - mS) = 1$. (Notice that this defines an *implicit* function for the optimum

value of S in terms of m and b. Unfortunately, we can't solve this equation for S directly. But Maple can solve it for us numerically.)

7. Make a graph of $H(S)$ when $b=3$ and $m=0.1$. $(0 \le S \le 20)$
8. Find the value of S that maximizes $H(S)$ when $b=3$ and $m=0.1$.
9. Graph the optimum value of S (call it S_{opt}, the value that maximizes $H(S)$) as a function of b $(0 < b \le 10)$ when $m=0.1$.
10. Graph the maximum harvest value ($H(S_{opt})$) as a function of b $(0 < b \le 10)$ when $m=0.1$.
11. Graph S_{opt} as a function of m $(0 < m \le 1)$ when $b=3$.
12. Graph $H(S_{opt})$ as a function of m $(0 < m \le 1)$ when $b=3$.
13. Graph S_{opt} as a function of b $(0 < b \le 10)$ and m $(0 < m \le 1)$.
14. Graph $H(S_{opt})$ as a function of b $(0 < b \le 10)$ and m $(0 < m \le 1)$.

Then, use the graphs that you produced and type up a paragraph or two describing how the birth rate (b) and mortality rate (m) parameters each affect the optimum number of spawners (S_{opt}) and the maximum harvest value ($H(S_{opt})$). What conclusions can you draw from this? Type those up as well.

What you need to turn in:

- Everything you produced with Maple (solved equations, answers, graphs, etc.). You can paste the graphs and equations into a Word document or you can just print out your Maple output.
- Your descriptions and conclusions in a Word document (or another word-processing format). You can also include your graphs and equations in that document.

Due Date: Tuesday, October 10, 2006