1) Let $u = 2x - 5$. Then $\frac{du}{dx} = 2$ and $dx = \frac{du}{2}$.

$$\int \frac{1}{(2x-5)^3}\,dx = \int \frac{1}{u^3}\cdot\frac{du}{2} = \frac{1}{2}\int u^{-3}du = \frac{1}{2}\cdot\frac{1}{-2}u^{-2} + C = -\frac{1}{4(2x-5)^2} + C$$

3) Let $u = 3x - 2$. Then $\frac{du}{dx} = 3$ and $dx = \frac{du}{3}$.

$$\int \sqrt{3x-2}\,dx = \int \sqrt{u}\cdot\frac{du}{3} = \frac{1}{3}\int u^{\frac{1}{2}}du = \frac{1}{3}\cdot\frac{1}{3/2}u^{\frac{3}{2}} + C = \frac{2}{9}(3x-2)^{\frac{3}{2}} + C$$

5) Let $u = 1 + x^5$. Then $\frac{du}{dx} = 5x^4$ and $dx = \frac{du}{5x^4}$.

$$\int \frac{x^4}{(1+x^5)^2}\,dx = \int \frac{x^4}{u^2}\cdot\frac{du}{5x^4} = \frac{1}{5}\int u^{-2}du = \frac{1}{5}\cdot\frac{1}{-1}u^{-1} + C = -\frac{1}{5(1+x^5)} + C$$

9) Let $u = 1 + e^x$. Then $\frac{du}{dx} = e^x$ and $dx = \frac{du}{e^x}$.

$$\int \frac{e^x}{1+e^x}\,dx = \int \frac{e^x}{u}\cdot\frac{du}{e^x} = \int u^{-1}du = \ln|u| + C = \ln|1+e^x| + C$$

13) Let $u = \ln x$. Then $\frac{du}{dx} = \frac{1}{x}$ and $dx = x\cdot du$.

$$\int \frac{\ln x}{x}\,dx = \int \frac{u}{x}\cdot x\cdot du = \int u\cdot du = \frac{1}{2}u^2 + C = \frac{1}{2}(\ln x)^2 + C$$

21) Let $u = 3 + x^2$. Then $\frac{du}{dx} = 2x$ and $dx = \frac{du}{2x}$.

$$\int_0^1 \frac{x}{(3+x^2)^3}\,dx = \int \frac{x}{u^3}\cdot\frac{du}{2x} = \frac{1}{2}\int u^{-3}du = \frac{1}{2}\cdot\frac{1}{-2}u^{-2} = \left(-\frac{1}{4(3+x^2)^2}\right)\Bigg|_0^1$$

$$= \left(-\frac{1}{4(3+1^2)^2}\right) - \left(-\frac{1}{4(3+0^2)^2}\right) = \left(-\frac{1}{64}\right) - \left(-\frac{1}{36}\right) = \frac{7}{576} = 0.012152\overline{7}$$

25) Let $u = 1 + \sqrt{x}$. Then $\frac{du}{dx} = \frac{1}{2}\cdot x^{-\frac{1}{2}} = \frac{1}{2}\cdot\frac{1}{x^{\frac{1}{2}}} = \frac{1}{2\sqrt{x}}$ and $dx = 2\sqrt{x}\cdot du$.

$$\int_1^4 \frac{(1+\sqrt{x})}{\sqrt{x}}\,dx = \int \frac{u}{\sqrt{x}}\cdot 2\sqrt{x}\cdot du = 2\int u\cdot du = 2\cdot\frac{1}{2}u^2 = (1+\sqrt{x})^2\Big|_1^4$$

$$= (1+\sqrt{4})^2 - (1+\sqrt{1})^2 = 3^2 - 2^2 = 5$$

27) Let $u = x - 2$. Then $\frac{du}{dx} = 1$ and $dx = du$ and $x = u + 2$.

$$\int \frac{x-1}{\sqrt{x-2}}\,dx = \int \frac{(u+2)-1}{\sqrt{u}}\,du = \int \frac{u+1}{u^{\frac{1}{2}}}\,du = \int \left(\frac{u^1}{u^{\frac{1}{2}}} + \frac{1}{u^{\frac{1}{2}}}\right)du = \int u^{\frac{1}{2}}du + \int u^{-\frac{1}{2}}du$$

$$= \frac{1}{3/2}u^{\frac{3}{2}} + \frac{1}{1/2}u^{\frac{1}{2}} + C = \frac{2}{3}(x-2)^{\frac{3}{2}} + 2\sqrt{x-2} + C$$

31) Let $u = \ln x$. Then $\frac{du}{dx} = \frac{1}{x}$ and $dx = x \cdot du$.

$$\int \frac{1}{x \ln x}\,dx = \int \frac{1}{x \cdot u} \cdot x \cdot du = \int u^{-1}du = \ln|u| + C = \ln|\ln x| + C$$