Math 152 - Sample Exam 1 - Spring 2005

Note: This sample exam is longer than the actual exam will be - it is designed to give you some additional practice problems.

1. Find the following limits, if they exist. If they don’t exist, state so.
   (a) \[
   \lim_{{y \to \infty}} \frac{4 - y}{2y + 3}
   \]
   (b) \[
   \lim_{{x \to 3}} \frac{4x + 2}{x + 1}
   \]
   (c) \[
   \lim_{{h \to 0}} \frac{h^2 - 2h}{3h^2 + h}
   \]
   (d) \[
   \lim_{{x \to -\infty}} \frac{5x^2}{2x^2 + 3x - 1}
   \]
   (e) \[
   \lim_{{x \to 2}} \frac{x^3 - 8}{x - 2}
   \]
   (f) \[
   \lim_{{y \to \infty}} \frac{3y}{2y + \sqrt{y}}
   \]
   (g) \[
   \lim_{{x \to -1}} \frac{8x^2 + x - 1}{x + 1}
   \]

2. Sketch the graphs of three different functions which are not continuous at \( x = 2 \), being sure to give the equations for the functions you are graphing.

3. For each of the following functions, state where the function is continuous:
   (a) \[
   f(x) = \begin{cases} 
   x + 1 & \text{for } x \leq -1 \\ 
   x^2 & \text{for } -1 < x \leq 1 \\ 
   x & \text{for } x > 1 
   \end{cases}
   \]
   (b) \[
   g(x) = \begin{cases} 
   x^2 & \text{for } x \leq 0 \\ 
   1/x^2 & \text{for } 0 < x < 1/2 \\ 
   x - 1 & \text{for } x \geq 1/2 
   \end{cases}
   \]

4. Find the derivative of each of the following functions:
   (a) \[
   f(x) = 3x^2 - x + 4/x
   \]
   (b) \[
   g(y) = \cos(1 - 3y)
   \]
   (c) \[
   y(t) = (t^2 + 4t)^5
   \]
   (d) \[
   f(z) = 3z \exp(z^2 - 4z)
   \]
   (e) \[
   g(x) = \ln(x^2 + 3x + 1)
   \]
   (f) \[
   f(t) = (3t - 1)/(t^2 + 1)
   \]

5. Find the equation of the line which is tangent to the graph of \( y = 3x^2 + 2x - 3 \) at \( x=1 \).
6. Suppose $L(t)$ gives the length of a fish in cm at time $t$, where $t$ is measured in months since hatching.
   (a) Give the definition of the derivative of $L(t)$ at time 2 months, $L'(2)$.
   (b) Explain in words what $L'(2)$ means, and give its units.
   (c) If $L(t)$ looks like the below graph, sketch a graph which indicates how $L'(t)$ changes from time 0 to time 10 months.

![Graph of L(t) vs t]

7. A particle’s position at time $t$ is given by $f(t) = (t^2 + 2t)^{1/2}$ where $t$ is measured in seconds and $f(t)$ is measured in meters. Give a function for the instantaneous speed of the particle and one for its acceleration. What is the particle’s speed at time 4 seconds? Is the particle accelerating or deccelerating at time 4 seconds? What is the velocity of the particle after a long time?

8. A reptile’s core body temperature in °C is found to vary through a day according to $T(t) = 20 + 10 \cos \left( \frac{\pi t}{12} \right)$ where $t$ is in hours and $t=0$ corresponds to noon.
   (a) Is the core temperature increasing or decreasing at 4PM?
   (b) What is the rate of change of body temperature at 6 PM?
   (c) At what times of day is the rate of change of core body temperature equal to zero?
1. (a) -1/2  (b) 7/2  (c) 1/3  (d) 5/2  (e) 12  (f) 3/2  (g) Doesn’t exist
2. Many possible choices, including f(x) = 1/(x-2), g(x)=ln(x-2),
y(x) = x^2 for x<2 and x for x ≥2
3. (a) Continuous on (-∞, -1) ∪ (-1, ∞)  (b) Continuous on (-∞, 0) ∪ (0, 5) ∪ (5, ∞)
4. (a) f′(x) = 6x - 1 - 4/x^2  (b) g′(y) = 3 sin(1 - 3y)  (c) y′(t) = 10(t + 2) (t^2 + 4t)^4
   (d) f′(z) = 3(2 z^2 - 4 z + 1) exp(z^2 - 4z)  (e) g′(x) = (2 x + 3) / (x^2 + 3x + 1)
   (f) f′(t) = (3+ 2t -3t^2) / (t^2 + 1)^2
5. slope = 8, point is (1,2), tangent line is y = 8x - 6
6. (a) L(t) = lim_{h→0} \frac{L(h+2) - L(2)}{h} = lim_{t→2} \frac{L(t) - L(2)}{t - 2}
   (b) L′(2) is the instantaneous rate of growth in length of a fish exactly at age 2 months.
   Its units are cm/month.
   (c)

7. v(t) = (t + 1)/ ( t^2 + 2t)^{1/2} , a(t) = -(t^2 + 2t)^{-3/2} , v(4) = 5/√24  m/s,
   decelerating at time t = 4 since a(4)<0, \lim_{t→∞} v(t) = 1 m/s
8. (a) T′(t) = -(5/6)π sin(πt/12) so T′(4) = -(5/6)π sin(π/3) = -(5√3) π/12 < 0 so temperature is decreasing at 4PM
   (b) T′(6) = -(5/6)π sin(π/2) = -(5/6) π °C/hr
   (c) T′(t) = 0 when t = 0 and t = 12 so temperature not changing at noon and midnight