Name______________________________

Early one morning, the police discovered the bodies of a man and a woman in the patio of a house. The temperature in the patio had been a constant 40°F since the night before. When the bodies were discovered, the temperature of the man's body was 65°F, and the temperature of the woman's body was 70°F. An hour later, the temperature of the man's body was 60°F, and the temperature of the woman's body was 65°F.

1) Assuming the man's body temperature was 98.6°F when he died, how long had he been dead when the police found the bodies (to the nearest tenth of an hour)? *(5 points)*

$$T(t) = T_0 + (T_1 - T_0)e^{-kt}$$

$$T(t) = 65 = 40 + (98.6 - 40)e^{-kt}$$
$$\Rightarrow 25 = 58.6e^{-kt}$$
$$\Rightarrow e^{-kt} = \frac{25}{58.6} \approx 0.4266$$
$$\Rightarrow \left(e^{-k}\right)^t = \frac{25}{58.6}$$

$$T(t+1) = 60 = 40 + (98.6 - 40)e^{-k(t+1)}$$
$$\Rightarrow 20 = 58.6\left(e^{-kt}\right)e^{-k}$$
$$\Rightarrow 20 = 58.6\left(\frac{25}{58.6}\right)e^{-k}$$
$$\Rightarrow e^{-k} = \frac{20}{25} = \frac{4}{5} = 0.8 \Rightarrow k = \ln\tfrac{5}{4} \approx 0.223$$

$$\Rightarrow \left(\frac{4}{5}\right)^t = \frac{25}{58.6} \Rightarrow \ln\left(\frac{4}{5}\right)^t = \ln\left(\frac{25}{58.6}\right) \Rightarrow t\ln\left(\frac{4}{5}\right) = \ln\left(\frac{25}{58.6}\right)$$
$$\Rightarrow t = \frac{\ln(25/58.6)}{\ln(4/5)} \Rightarrow t \approx 3.8 \text{ hours}$$

2) Assuming the woman died at the same time as the man, what was her body temperature when she died (to the nearest tenth of a °F)? *(5 points)*

$$T(t) = 70 = 40 + (T_1 - 40)e^{-kt}$$
$$\Rightarrow 30 = (T_1 - 40)e^{-kt}$$
$$\Rightarrow e^{-kt} = \frac{30}{(T_1 - 40)}$$
$$\Rightarrow (T_1 - 40) = \frac{30}{e^{-kt}}$$

$$T(t+1) = 65 = 40 + (T_1 - 40)e^{-k(t+1)}$$
$$\Rightarrow 25 = (T_1 - 40)\left(e^{-kt}\right)e^{-k}$$
$$\Rightarrow 25 = (T_1 - 40)\left(\frac{30}{(T_1 - 40)}\right)e^{-k}$$
$$\Rightarrow e^{-k} = \frac{25}{30} = \frac{5}{6} \Rightarrow k = \ln\tfrac{6}{5} \approx 0.182$$

$$\Rightarrow T_1 = \frac{30}{\left(e^{-k}\right)^t} + 40 \Rightarrow T_1 = \frac{30}{(5/6)^{3.8}} + 40 \Rightarrow T_1 \approx 100.0^\circ$$