DO ALL YOUR WORK AND WRITE ALL YOUR ANSWERS ON THE ADDITIONAL PAPER THAT IS PROVIDED. YOU MUST SHOW YOUR WORK IN ORDER TO RECEIVE FULL CREDIT. PLEASE USE ONLY ONE SIDE OF EACH SHEET AND WRITE YOUR NAME ON EACH SHEET. SHOW AS MUCH WORK AS POSSIBLE BECAUSE YOU MAY RECEIVE PARTIAL CREDIT FOR THE WORK YOU DO IF YOUR ANSWER IS INCORRECT. CIRCLE YOUR ANSWERS.

EACH REGULAR PROBLEM IS WORTH 25 POINTS. THE BONUS IS WORTH 10 POINTS.

ROUND DECIMAL ANSWERS TO THREE SIGNIFICANT DIGITS.
(EX. 45.6, 0.00456, 4.56%, 45.6%, 0.456%, etc.)

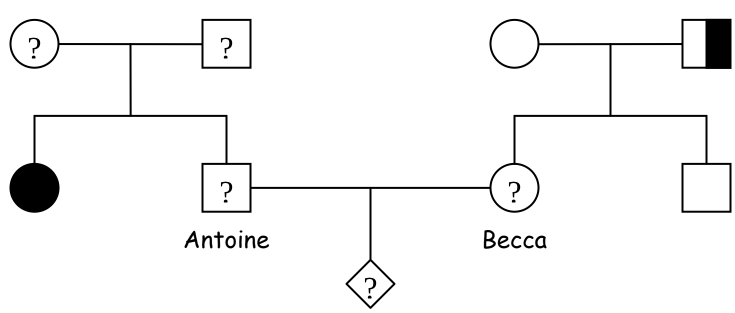
1. A couple, both of whom are Tay-Sachs carriers, decides to have three children.
   a. What is the probability that <u>at least one</u> of their three children will have Tay-Sachs?
   b. If none of their three children has Tay-Sachs, what is the probability that <u>at least one</u> of the children is a Tay-Sachs carrier?

HINT: Break the problem down into smaller parts first and assume that each child's genotype is independent of the other children's genotypes.

2. One day in an emergency room, 25% of the patients had a life-threatening injury, 35% of the patients required an X-ray, and 45% of the patients neither had a life-threatening injury nor required an X-ray.
   a. What percentage of the patients required an X-ray but did not have a life-threatening injury?
   b. What percentage of the patients had a life-threatening injury but did not require an X-ray?
   c. Which group of patients was more likely to require an X-ray: those with a life-threatening injury or those without a life-threatening injury? (or was it the same for both groups?) Justify your answer.

HINT: Use a Venn Diagram and use actual numbers of patients.

3. Consider the following pedigree for a family with a history of cystic fibrosis, which is a recessive genetic disease. Neither Antoine nor Becca has cystic fibrosis. Also, neither of Antoine's parents has cystic fibrosis.



   a. What is the probability that Antoine carries the cystic fibrosis allele?
   b. What is the probability that Becca carries the cystic fibrosis allele?
   c. What is the probability that Becca and Antoine's child will have cystic fibrosis?

4. The standard test for colorectal cancer is the hemoccult test. The incidence of colorectal cancer among adults over 50 years old is 0.3%. 50% of people with colorectal cancer will have a positive hemoccult test, but 3% of people who do not have colorectal cancer will also have a positive hemoccult test. A doctor performs a hemoccult test for a 51-year old patient as part of a routine checkup.
   a. If the patient's test is positive, what is the probability that the patient actually has colorectal cancer?
   b. If the patient's test is negative, what is the probability that the patient <u>does not have</u> colorectal cancer?
   c. Based on the patient's family history, the doctor determines that the patient has a 50% prior probability of having colorectal cancer. Now if the patient's test is positive, what is the probability that the patient actually has colorectal cancer?

HINT: USE A TREE DIAGRAM AND USE ACTUAL NUMBERS OF PATIENTS.

5. The following table summarizes the number of successes and failures for low risk and high risk surgeries at two hospitals:

	Low Risk		High Risk	
	Success	Failure	Success	Failure
Hospital A	360	140	160	640
Hospital B	330	170	20	180

   a. What was the success rate for high risk surgeries at Hospital A?
   b. What was the success rate for high risk surgeries at Hospital B?
   c. What was the overall success rate for surgeries at Hospital A?
   d. What was the overall success rate for surgeries at Hospital B?
   e. What fraction of surgeries performed at Hospital A were high risk?
   f. What fraction of surgeries performed at Hospital B were high risk?
   g. Which hospital is more likely to publish this data in <u>aggregated</u> form (that is, without breaking it down by risk)?
   h. A friend just had a successful high risk surgery. What is the probability that it was performed at Hospital A?
   i. Another friend just had a successful surgery at Hospital B. What is the probability that it was low risk?

6. In a long-term study of breast cancer patients, some patients received radiation therapy after having their tumors removed, while others did not. Of the patients who did not receive radiation therapy, 80% were still cancer-free after 4 years and 64% were still cancer-free after 20 years.
   a. Of the non-radiation patients who were still cancer-free after 4 years, what fraction was still cancer-free after 20 years?
   b. Of the non-radiation patients who had a recurrence of cancer, what fraction had their recurrence within 4 years of having their original tumors removed?

HINT: USE A VENN DIAGRAM AND USE ACTUAL NUMBERS OF PATIENTS.

BONUS: Of the patients who did receive radiation therapy, 83% were still cancer-free after 20 years. Overall, 79.2% of the patients in the entire study were cancer-free after 20 years. What fraction of the patients in the study received radiation therapy?

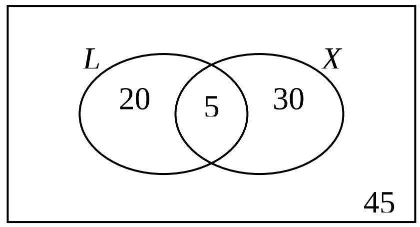
Answers to the Regular Problems:

1.

a. Let T be the event: "at least one of the three children will have Tay-Sachs." Then, $\overline{T}$ is the event: "none of the three children will have Tay-Sachs." Let T_1, T_2, and T_3 be the events of having Tay-Sachs for child 1, 2, and 3, respectively. Then, $\overline{T} = \overline{T}_1 \cap \overline{T}_2 \cap \overline{T}_3$ and since T_1, T_2, and T_3 are independent of each other, then $P\left(\overline{T}\right) = P\left(\overline{T}_1\right)P\left(\overline{T}_2\right)P\left(\overline{T}_3\right)$, and thus $P\left(T\right) = 1 - P\left(\overline{T}\right) = 1 - P\left(\overline{T}_1\right)P\left(\overline{T}_2\right)P\left(\overline{T}_3\right)$. Since both parents are Tay-Sachs carriers, we know that for each child, the probability of not having Tay-Sachs is $\frac{3}{4}$ (see the Punnett Square for example 59.2 on page 672 of the textbook). Thus, $P\left(\overline{T}_1\right) = P\left(\overline{T}_2\right) = P\left(\overline{T}_3\right) = \frac{3}{4}$ and
$P\left(T\right) = 1 - \left(\frac{3}{4}\right)\left(\frac{3}{4}\right)\left(\frac{3}{4}\right) = 1 - \left(\frac{3}{4}\right)^3 = 1 - \frac{27}{64} = \frac{37}{64} = 0.578$.

b. Let C be the event: "at least one of the three children is a Tay-Sachs carrier." Then, $\overline{C}$ is the event: "none of the three children is a Tay-Sachs carrier." Let C_1, C_2, and C_3 be the events of being a Tay-Sachs carrier for child 1, 2, and 3, respectively. Then, $\overline{C} = \overline{C}_1 \cap \overline{C}_2 \cap \overline{C}_3$ and since C_1, C_2, and C_3 are independent of each other, then $P\left(\overline{C}\right) = P\left(\overline{C}_1\right)P\left(\overline{C}_2\right)P\left(\overline{C}_3\right)$, and thus
$P\left(C\right) = 1 - P\left(\overline{C}\right) = 1 - P\left(\overline{C}_1\right)P\left(\overline{C}_2\right)P\left(\overline{C}_3\right)$. Since both parents are Tay-Sachs carriers, and since none of the children have Tay-Sachs, then we know that for each child, the probability of being a Tay-Sachs carrier is $\frac{2}{3}$ (see example 59.2 on page 672 of the textbook) and the probability of not being a carrier is $\frac{1}{3}$. Thus,
$P\left(\overline{C}_1\right) = P\left(\overline{C}_2\right) = P\left(\overline{C}_3\right) = \frac{1}{3}$ and
$P\left(C\right) = 1 - \left(\frac{1}{3}\right)\left(\frac{1}{3}\right)\left(\frac{1}{3}\right) = 1 - \left(\frac{1}{3}\right)^3 = 1 - \frac{1}{27} = \frac{26}{27} = 0.963$.

2. See the Venn diagram below for this problem. For simplicity, we will assume that there were a total of 100 patients. That means that 25 patients had a life-threatening injury (L), and 35 patients required an X-ray (X). 45 patients did not have a life-threatening injury and also did not need an X-ray $(\overline{L} \cap \overline{X})$. That means that 55 patients either had a life-threatening injury or required an X-ray or both (since 100 – 45 = 55) $(L \cup X)$.

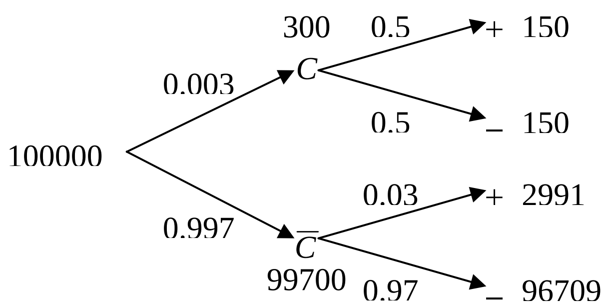


   a. Of those 55 patients, 25 had a life-threatening injury, which means that 30 patients did not have a life-threatening injury but they still required an X-ray. Thus, $P(X - L) = \frac{30}{100} = 0.3$
   b. Also, of those 55 patients, 35 required an X-ray, which means that 20 patients did not require an X-ray but they still had a life-threatening injury. Thus, $P(L - X) = \frac{20}{100} = 0.2$
   c. Of the 25 patients that had a life-threatening injury, 5 also required an X-ray, which represents 20% of those patients. $P(X \mid L) = \dfrac{P(X \cap L)}{P(L)} = \dfrac{5}{25} = 0.2 = 20\%$

      Of the 75 patients that did not have a life-threatening injury (since 100 – 25 = 75), 30 also required an X-ray, which represents 40% of those patients.
      $$P(X \mid \overline{L}) = \frac{P(X \cap \overline{L})}{P(\overline{L})} = \frac{P(X - L)}{P(\overline{L})} = \frac{30}{75} = 0.4 = 40\%$$
      Thus, patients without a life-threatening injury were more likely to require an X-ray.

3. We will let c represent the cystic fibrosis disease allele and let C represent the normal allele. Since Antoine's parents do not have cystic fibrosis but one of their children does, that means they must both be carriers (Cc) (see example 59.2 on page 672 of the textbook).
   a. Since Antoine does not have cystic fibrosis, the probability that he carries the cystic fibrosis allele is $\frac{2}{3}$ (see example 59.2 on page 672 of the textbook).
   b. From the pedigree diagram, Becca's mother's genotype is CC, and Becca's father's genotype is Cc. Thus, Becca could only have inherited a C allele from her mother, and she had a 50% probability of inheriting a c allele from her father, so the probability that Becca carries the cystic fibrosis allele is $\frac{1}{2}$.
   c. In order for Becca and Antoine's child to have cystic fibrosis, both Becca and Antoine would have to be carriers (Cc) (see example 59.8 on pages 676-677 of the textbook). If Becca and Antoine are both carriers, then the probability that their child would have cystic fibrosis is $\frac{1}{4}$. Since the probability of Becca being a carrier is independent of Antoine being a carrier, the probability that both of them are carriers is $\frac{1}{2} \cdot \frac{2}{3} = \frac{1}{3}$. If we let C be the event: "their child has cystic fibrosis," and let D be the event: "both Becca and Antoine are carriers," then the probability that their child has cystic fibrosis is $\frac{1}{12}$ since $P(C) = P(C \cap D) = P(C \mid D)P(D) = \frac{1}{4} \cdot \frac{1}{3} = \frac{1}{12}$.

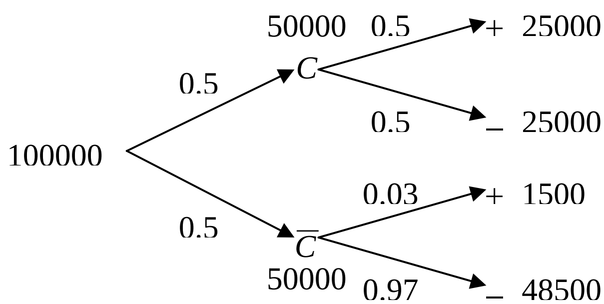
4. For simplicity, we will assume that 100,000 people are taking the same test as the patient. Then, we let C be the event: "has colorectal cancer," $+$ be the event: "has a positive hemoccult test," and $-$ be the event: "has a negative hemoccult test." Then, based on the information given in the problem: $P(C)=0.003$, $P(\overline{C})=0.997$, $P(+\mid C)=0.5$, $P(-\mid C)=0.5$, $P(+\mid \overline{C})=0.03$, and $P(-\mid \overline{C})=0.97$. Putting this information together yields the following tree diagram:



a. $P(C\mid +)=\dfrac{P(C\cap +)}{P(+)}=\dfrac{150}{150+2991}=\dfrac{150}{3141}=0.0478$

b. $P(\overline{C}\mid -)=\dfrac{P(\overline{C}\cap -)}{P(-)}=\dfrac{96709}{96709+150}=\dfrac{96709}{96859}=0.998$

c. For this part, $P(C)=0.5$ and $P(\overline{C})=0.5$, which yields the following tree diagram:



Thus, in this case, $P(C\mid +)=\dfrac{P(C\cap +)}{P(+)}=\dfrac{25000}{25000+1500}=\dfrac{25000}{26500}=0.943$

5. The answers to this problem can be calculated directly from the table.

a. $\frac{160}{160+640}=\frac{160}{800}=0.2=20\%$

b. $\frac{20}{20+180}=\frac{20}{200}=0.1=10\%$

c. $\frac{360+160}{360+140+160+640}=\frac{520}{1300}=0.4=40\%$

d. $\frac{330+20}{330+170+20+180}=\frac{350}{700}=0.5=50\%$

e. $\frac{160+640}{360+140+160+640}=\frac{800}{1300}=\frac{8}{13}=0.615=61.5\%$

f. $\frac{20+180}{330+170+20+180}=\frac{200}{700}=\frac{2}{7}=0.286=28.6\%$

g. Hospital B (since 50% > 40%, but 10% < 20%)

h. $\frac{160}{160+20}=\frac{160}{180}=\frac{8}{9}=0.889=88.9\%$

i. $\frac{330}{330+20}=\frac{330}{350}=\frac{33}{35}=0.943=94.3\%$

6. For simplicity, we will assume that there were a total of 100 patients who did not receive radiation therapy. Of these, 80 patients were still cancer-free after 4 years, which means that 20 patients had a recurrence of cancer within the first 4 years. After 20 years, 64 patients were still cancer-free, which means that within the 20 years, a total of 36 patients had a recurrence of cancer.
    a. Of the 80 patients who where still cancer-free after 4 years, $\frac{64}{80} = \frac{4}{5} = 0.8 = 80\%$ were still cancer-free after 20 years.
    b. Of the 36 patients who had a recurrence of cancer, $\frac{20}{36} = \frac{5}{9} = 0.556 = 55.6\%$ had their recurrence within 4 years of having their original tumors removed.