1. For the following linear programming problem:

Minimize $C = 10x + 30y$

Subject to $\begin{cases} x + 2y \ge 12 \\ 3x - 2y \le 12 \\ x \ge 0, y \ge 0 \end{cases}$

- Draw a graph of the system of inequalities. Make sure to shade in the feasible region.
- On the graph, label each of the boundary lines of the feasible region with its equation.
- On the graph, label each of the vertices of the feasible region with its coordinates.
- State whether the feasible region is "bounded" or "unbounded."
- If the problem has a solution, state the values of x, y, and C for the solution.

  If the problem does not have a solution, write "no solution."

2. The city runs an after-school program for children aged 6 to 10 and hires college students and high school students to supervise them. College students each supervise 12 children and are paid $10 an hour. High school students each supervise 8 children and are paid $6 an hour. By regulation, the city must hire at least as many college students as high school students. If the city is willing to spend no more than $240 per hour on payroll, what is the maximum number of children that can attend the after-school program? How many college students and how many high school students should the city hire to achieve that maximum?

   If the city is willing to spend an additional $80 per hour on payroll, how many additional children could attend the after-school program?

3. For the following initial simplex tableau:

	x_1	x_2	x_3	s_1	s_2	
s_1	0	1	1	1	0	7
s_2	1	1	−1	0	1	6
P	−5	−4	−3	0	0	0

   a. Write the standard maximum problem that corresponds to the tableau.
   b. Solve the problem using the simplex method. Make sure to write out each tableau in the process and circle the pivot element in each tableau.
      If the problem has a solution, state the values of x_1, x_2, x_3, and P for the solution.
      If the problem does not have a solution, write "no solution."

4. For the following minimum problem:

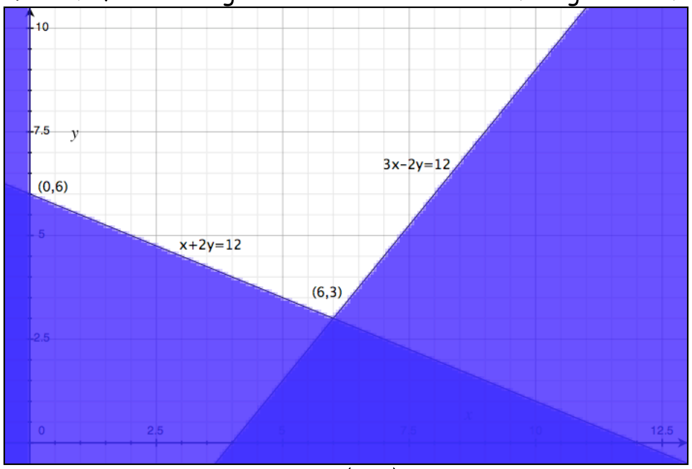
Minimize $C = 3y_1 + 4y_2 + 10y_3$

Subject to $\begin{cases} y_1 - y_2 - y_3 \le 10 \\ y_1 - y_2 - 2y_3 \ge 20 \\ y_1 \ge 0, y_2 \ge 0, y_3 \ge 0 \end{cases}$

   Solve the problem by finding its dual maximum problem and using the simplex method.
   (You don't need to explicitly write out the dual maximum problem.)
   Make sure to write out each tableau in the process and circle the pivot element in each tableau.
   If the problem has a solution, state the values of y_1, y_2, y_3, and C for the solution.
   If the problem does not have a solution, write "no solution."
   (NOTE: The original minimum problem might not be in standard form.)

Answers:

1. The feasible region is unbounded and is the region in white in the figure below.



Solution: $C = 150$ at $(6,3)$

2. The maximum number of children that can attend is 300. The city should hire 15 college students and 15 high school students.
   If the city is willing to spend an additional $80 per hour on payroll, 100 additional children could attend the after-school program.

3.

   a. Minimize $P = 5x_1 + 4x_2 + 3x_3$

   Subject to $\begin{cases} x_2 + x_3 \le 7 \\ x_1 + x_2 - x_3 \le 6 \\ x_1 \ge 0, x_2 \ge 0, x_3 \ge 0 \end{cases}$

   b. $x_1 = 13, x_2 = 0, x_3 = 7, P = 86$

4. no solution