1. For the following system of equations: $\begin{cases} 4x_1 + 6x_2 + 5x_3 + 9x_4 = 75 \\ 4x_1 + x_2 + 2x_3 + 4x_4 = 10 \\ x_1 + 4x_2 + 3x_3 + 6x_4 = 65 \\ 4x_1 + 3x_2 + 3x_3 + 4x_4 = 10 \end{cases}$.

   a. Write the system as an augmented matrix.
   b. Write the system as a matrix equation in the form $AX = B$. (Make sure to write out each matrix explicitly.)
   c. Given that $A^{-1} = \begin{pmatrix} 2 & -0.2 & -2.2 & -1 \\ 4 & -1 & -4 & -2 \\ -8 & 1 & 8 & 5 \\ 1 & 0.2 & -0.8 & -1 \end{pmatrix}$, find the solution to the system.

2. For the following augmented matrix: $\begin{pmatrix} 1 & 5 & 8 & | & 10 \\ 1 & 4 & 6 & | & 9 \\ 1 & 3 & 4 & | & 8 \end{pmatrix}$

   a. Write the system of equations that corresponds to the matrix.
   b. Row-reduce the matrix **making sure to clearly show each step**.
   c. If the system has a single solution, state the solution. If the system is inconsistent, write "inconsistent." If the system is dependent, write the solution set in <u>parameterized</u> form.

3. The city runs an after-school program at two locations for children aged 6 to 10 and hires college students and high school students to supervise them. College students each supervise 12 children and are paid $10 per hour. High school students each supervise 7 children and are paid $6 per hour. The number of children and the hourly payroll at each location are given in the table. How many college students and how many high school students work at each location?

   |  | Location A | Location B |
   |---|---|---|
   | Children | 90 | 119 |
   | Hourly Payroll | $76 | $100 |

4. The concession stand at a movie theater offers the following combos:
   1 large popcorn, 1 large drink, 1 small popcorn, and 2 small drinks for $17
   1 large popcorn, 1 large drink, and 4 small drinks for $18
   2 large popcorns, 1 large drink, and 2 small drinks for $19
   A small drink by itself costs $2.

   a. How much does 1 small popcorn cost?
   b. How much does 1 large popcorn cost?
   c. If a large drink by itself costs $4, is it better to buy a large drink separately or as part of a combo? Justify your answer.

**ANSWERS:**

1.

a.
$$\begin{pmatrix} 4 & 6 & 5 & 9 & | & 75 \\ 4 & 1 & 2 & 4 & | & 10 \\ 1 & 4 & 3 & 6 & | & 65 \\ 4 & 3 & 3 & 4 & | & 10 \end{pmatrix}$$

b.
$$\begin{pmatrix} 4 & 6 & 5 & 9 \\ 4 & 1 & 2 & 4 \\ 1 & 4 & 3 & 6 \\ 4 & 3 & 3 & 4 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 75 \\ 10 \\ 65 \\ 10 \end{pmatrix}$$

c.
$$X = A^{-1}B = \begin{pmatrix} 2 & -0.2 & -2.2 & -1 \\ 4 & -1 & -4 & -2 \\ -8 & 1 & 8 & 5 \\ 1 & 0.2 & -0.8 & -1 \end{pmatrix} \begin{pmatrix} 75 \\ 10 \\ 65 \\ 10 \end{pmatrix} = \begin{pmatrix} -5 \\ 10 \\ -20 \\ 15 \end{pmatrix} \quad \begin{cases} x_1 = -5 \\ x_2 = 10 \\ x_3 = -20 \\ x_4 = 15 \end{cases}$$

2.

a.
$$\begin{cases} x_1 + 5x_2 + 8x_3 = 10 \\ x_1 + 4x_2 + 6x_3 = 9 \\ x_1 + 3x_2 + 4x_3 = 8 \end{cases}$$

b.
$$\begin{pmatrix} 1 & 0 & -2 & | & 5 \\ 0 & 1 & 2 & | & 1 \\ 0 & 0 & 0 & | & 0 \end{pmatrix}$$

c.
$$\begin{cases} x_1 = 2t + 5 \\ x_2 = 1 - 2t \\ x_3 = t \end{cases}$$

3.  Location A: 4 college students and 6 high school students
    Location B: 7 college students and 5 high school students

4.
a. $3
b. $5
c. A large drink in a combo costs $5, so it is better to buy it separately.