

Homework
UTK – M448 – Honors Advanced Calculus II – Spring 2016
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1. Prove by hand that $f : \mathbb{R}^2 \rightarrow \mathbb{R}$, $(x, y) \mapsto x^2 + 2xy^2 + 5y^3$ is differentiable at $(2, 1)$ with $DF(2, 1) = [6, 3]$. Note: $\frac{\partial f}{\partial x}(2, 1) = 2x + 2x^2|_{x=2, y=1} = 6$ and $\frac{\partial f}{\partial y}(2, 1) = 4xy + 15y^2|_{x=2, y=1} = 23$.

2. Show that the operator norm $\|\cdot\|$ is a norm.

3. Show for $\mathbb{R}^m, \mathbb{R}^n$ equipped with $\|\cdot\|_1$ (where $\|\vec{x}\|_1 := \sum_i |x_i|$) that the operator norm $\|A\|$

of $A = ((a_{ij})) = \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & & \vdots \\ a_{m1} & \cdots & a_{mn} \end{bmatrix}$ is

$$\|A\| = \max_{j=1}^n \sum_{i=1}^m |a_{ij}|$$

(i.e., the largest 1-norm of any column).