Homework
UTK – M431 – Differential Equations
Spring 2003, Jochen Denzler, MWF 11:15–12:05, BU 475

21. see manuscript on Linear Systems

22. see manuscript on Linear Systems

23. see manuscript on Linear Systems

24. Find the general solution for the system

$$\begin{aligned} x_1' &= 2x_1 - 3x_2 \\ x_2' &= x_1 - 2x_2 \ . \end{aligned}$$

Draw some of these solutions in the phase space (i.e., draw the parametrized curves

$$\begin{aligned} x_1(t) &= \ldots \text{ (the solution you found)} \\ x_2(t) &= \ldots \text{ (the solution you found)} \end{aligned}$$

Find those solutions that satisfy $x(t) \to 0$ as $t \to \infty$ and be sure to include them in the figure. Also find those solutions that satisfy $x(t) \to 0$ as $t \to -\infty$ and include them, too. Is the equilibrium $x = 0$ stable?

25. The same questions as before for the system

$$\begin{aligned} x_1' &= x_1 + 5x_2 \\ x_2' &= -x_1 - x_2 \ . \end{aligned}$$

26. We have learnt that for a diagonal matrix (or also a triangular matrix) the eigenvalues are just the diagonal elements. For other matrices this is typically not true. However, the following facts from matrix algebra are true for every square matrix, and useful to know: The *sum* of all eigenvalues of a matrix equals the *sum* of its diagonal elements (this sum is also called the trace of the matrix). The product of all eigenvalues is the determinant. If you don't know or remember these facts from M200, commend them to your memory now, and use them to decide the following question in one line:

Is the equilibrium $x = y = z = 0$ of the system

$$\begin{aligned} x' &= x + y + z \\ y' &= -x + y - z \\ z' &= x - y + z \end{aligned}$$

stable?