Homework
UTK – M431 – Differential Equations
Spring 2003, Jochen Denzler, MWF 11:15–12:05, BU 475

5. See notes on the Frobenius method, page 4-5

6. Some training problems:

(a) Find the general solution of the ODE

$$(1 - x - x^2)y'' - (4x + 2)y' - 2y = 0$$

in terms of power series. In particular, make sure to write down the general recursion formula for the coefficients. You will get a nice recursion formula, but you may not be able to come up with a closed formula for a_n.

(Test to check your calculation: If you choose $a_0 = a_1 = 1$, you will obtain $a_6 = 13$)

What can you conclude about the radius of convergence of the resulting Taylor series?

(b) Same question with

$$(1 - x)y'' + xy' - y = 0$$

You will again get a recursion formula, but you may not be able to come up with a closed formula for a_n.

Check that $a_n = 1/n!$ is one solution of this recursion.

Now find the special solution that also satisfies the initial conditions $y(0) = 0$, $y'(0) = 1$; you will be able to go all the way up to a (very simple) closed formula for this solution y.

(c) Find the general solution of the ODE

$$2y'' + xy' + y = 0$$

This time, after finding the recursion formula, you should be able to come up with a closed formula for the Taylor coefficients. If you are really good at eyeballing the Taylor series I gave you on the 1st page of the power series manuscript, you will even be able to get an explicit formula for the solutions (i.e., an explicit summation of the power series), but that's an embellishment of lesser importance.

(d) Find the general solution of the ODE

$$y'' - x^2y' - xy = 0$$

Again, after finding the recursion formula, you will be able to come up with a closed formula for the Taylor coefficients. But don't attempt to actually sum the series in terms of elementary functions