Homework
UTK – M431 – Differential Equations
Spring 2003, Jochen Denzler, MWF 11:15–12:05, BU 475

1. Write the expression $\frac{1}{(1-x)(1+x)^2}$ as a power series centered at 0, calculated explicitly up to the order x^6.

2. Let $y(x)$ be the solution to the IVP $y' = \cos x + xy$, $y(0) = 1$. Calculate $y'(0)$, $y''(0)$, and $y'''(0)$. (We'll discuss two different ways of doing this and compare them.) — Also explain why the question to calculate $y'(1)$, $y''(1)$, and $y'''(1)$ would be much more difficult to answer (actually prohibitively difficult without a computer).

3. (The following is a nice training problem for power series, even though it does not relate to differential equations.) An unknown function f, given by a convergent (for sufficiently small x) power series

$$f(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + \dots$$

satisfies the equation

$$f(x) = 1 + \frac{x}{6}\left(f(x)^3 + 3f(x)f(x^2) + 2f(x^3)\right) .$$

Calculate a_0, a_1, a_2, a_3, a_4, a_5, a_6.

4. (Everybody) Write the solution to the IVP

$$y'' = xy\ , \quad y(0) = 1\ ,\ y'(0) = 0$$

as a power series. (We'll share this part of the job:) Use a pocket calculator, together with the power series method to calculate the solution, to 4 digits behind the decimal point, at $x = 0, 0.25, 0.5, 0.75, 1, 1.25, 1.5, 1.75, 2$ and $x = -0.25, -0.5, -0.75, \dots -3.75, -5$. Also record how many terms in the series you needed to include, in order to achieve the required accuracy. (Heuristic convergence check sufficient for our purposes.)